Mathematics 355 : Homework # 1

Due (in class): Monday, January 28.

The first 8 problems are from the text:

1. pgs. 37-46: # 1, 5 (a) and (b), 7, 11, 16, 21, 35, 37.

2. Let $\omega \in I = [0,1]$. Show that ω can be written in the form $\sum_{j=1}^{\infty} \frac{a_i}{2^i}$ with $a_j = 0, 1$. Show that this expansion is unique when we restrict to nonterminating series.

3. Let $f : [0,1] \to \mathbf{R}$ be a non-negative monotone function on the unit interval. Prove Chebyshev's inequality:

$$m(\{\omega \in I; f(\omega) > \alpha\}) < \frac{1}{\alpha} \int_0^1 f dx$$

with the integral on the right being the Riemann integral.

4. A gambler has an initial stake of one dollar. Calculate the probability of ruin at times 1, 3 and 5. Show that the chance of eventual ruin is at least 70%.