``Caïssan magic squares" by ``Ursus"

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6 August 1881, p. 142 (part 1/3)

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CAISSAN MAGIC SQUARES.

By "URSUS."

ONCE UPON A TIME, when Orpheus was a little boy, long before the world was blessed with the "Eastern Question," there dwelt in the Balkan forests a charming nymph by name Caïssa. The sweet Caïssa roamed from tree to tree in Dryad meditation fancy free. As for trees, she was, no doubt, most partial to the box and the ebony. Mars read of her charms in a "society" paper, saw her photograph, and started by the first express for Thracia. He came, he saw, but he conquered not—in fact, was ingloriously repulsed. As he was wandering by the Danube blurting out, to give vent to his chagrin, many unparliamentary expressions, a Naiad advised him to go to Euphron, who ruled over a sort of Lowtherian Arcadia. Euphron good-naturedly produced from his stores a mimic war game, warranted to combine amusement with instruction. The warrior—in mufti—took the game to Caïssa, taught her how to play, called it after her name, and, throwing off his disguise, proposed. The delighted Caïssa consented to become Mrs Mars. He spent the honeymoon in felling box trees and ebony ditto, she in fashioning the wood to make tesselated boards and quaint chessmen to send all over the uncivilised world.

Such is our profane version of the story, told so prettily in Ovidian verse by Sir William Jones, the Orientalist. Such is Caïssa, as devoutly believed in by Phillidorians as though she were in Lempriere.

Our Caïssa, however, shall herself be a chessman—we beg her pardon—chesswoman. Like the "queen" in the Russian game, she shall have the power of every man, moving as a king, rook, bishop, or pawn (like the queen in our game), and also as a knight.

Caïssa then is complete mistress of the martial board. In Fig. A^1, suppose her to be at R; she can as a rook move in the direction r^1, r^2, r^3, or r^4. If at B in Fig. A^2, she can move, as a bishop, in

		r^1		
r^4		R		r^2
		r^3		

Fig. A^1

b^1				b^2
		B		
b^4				b^3

Fig A^2.

the direction b^1, b^2, b^3, or b^4. In Fig. B^1, suppose her to be at K; she can, as a knight, leap to k^1, k^2, k^3, k^4, k^5, k^6, k^7, or k^8.

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These sixteen moves may conveniently be indicated by the points

	k^2		k^3	
k^1				k^4
		K		
k^8				k^5
	k^7		k^6	

Fig. B[1]

NW	NNW	N	NNE	NE
WNW				ENE
W		C		E
WSW				ESE
SW	SSW	S	SSE	SE

Fig. B[2]

of the compass as in Fig. B[2]. The rook moves E, W, N, or S. The bishop, NE, NW. SE, or SW. The knight, WNW, NNW, NNE, ENE, ESE, SSE, SSW, or WSW. Caïssa can go in any one of these directions.

We must now describe what is meant by a *path*. A magic square is most conveniently named by the number of cells in one of its sides. Hence the ordinary chessboard of sixty-four cells is an eight-square; a square with eighty-one cells is a nine-square, and so on. A path in an eight-square consists of eight cells; a path in a nine-square consists of nine cells, and so on; to speak generally, a path contains as many cells as the side of the square.

When Caïssa is following a path, she must keep on moving or leaping in the same direction to complete the required number of cells. Thus, if she starts on a north rook-path she must complete the required number of cells in that path; so, if she commences an east-nor'-east knight-path, she must leap the required number of cells in that path.

Lastly, and this is the only difficulty, it must be assumed that the chess board or eight-square on which Caïssa starts has chessboards or eight-squares similar to itself contiguous on every side. This is partly shown in Fig. C. The thick line surrounding 64 small

							6								6	
				5								5				
	4								4							
						3										
			2													
A								A								A
x					8	q	h		•					q		
w		7		p		g				•		p				
v		o			f					o	•					
n				e				n				•				
t			d			m							•			
s		c		l										•		
r	b	k													•	
A	r	s	t	u	v	w	x	A	•	•	•	•	•	•	•	A
x						q	h									
w				p		g										

Fig. C.

squares (called cells when speaking of magic squares), in the lower left-hand or SW. portion of the figure, represents, say, a chessboard. The similar figures above and to the right represent three imaginary chessboards contiguous to the first, and the one column outside the thick line to the extreme right represents a portion of other chessboards, the remainder not being shown for want of space. There might also be imaginary boards to the left and below, but they are not necessary for our present illustration of Caïssa's moves.

Assume that Caïssa is placed in the cell marked A in the lower left-hand board, which, for distinction, we shall call the *home board*, and that she starts on her paths from there. A*r* to *x* is a rook-path; A*b* to *h*, a bishop-path; A*k* to *q*, a knight-path. Other rook and bishop paths are indicated by dots in the imaginary boards; and a path of another kind, called Caïssa's special path, is shown by the numerals in the upper boards, starting from A in the upper left-hand board (which stands for 1) and going to 2, 3, 4, 5, 6. The completion of this path above and beyond 6 is omitted for want of space. Caïssa has at least two special paths in a perfect Caïssan magic square.

Now, calling A the *initial cell*, it will be observed that when Caïssa has completed an eight-cell path on the *home board*, a one-cell move brings her to the initial cell of one of the imaginary contiguous boards. When, however, she cannot complete such a path on the home board by continuing in the direction in which she started, she must complete it in those cells of the *home board* which are *similarly situated* to those in the imaginary boards in which the path, when continued, would fall. So, with a numerical series, the terms falling in the cells of contiguous boards must be transferred to corresponding cells of the *home board*. This is shown by the thin-faced letters in Fig. C.

From the above considerations we may see that a path can be

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From the above considerations we may see that a path can be completed by working, so to speak, *backwards*, and that, though Caïssa has, including her special paths, twenty *initial moves* from any given cell, she has only ten paths, for a North-path is identical with a South-path, a North-east with a South-west, and so on. But as there are in an eight-square sixty-four starting points, or eight in any row, she has in that square *eighty distinct paths*, each of which, when the square is properly filled with numbers, gives a constant summation. If the numbers 1 to 64 are chosen, as seems most natural, the constant summation of each path will be 260.

28	a	B	c	D	e	F	g	H
61	1	58	3	60	8	63	6	61
52	16	55	14	53	9	50	11	52
45	17	42	19	44	24	47	22	45
36	32	39	30	37	25	34	27	36
5	57	2	59	4	64	7	62	5
12	56	15	54	13	49	10	51	12
21	41	18	43	20	48	23	46	21
28	40	31	38	29	33	26	35	28
61	A	b	C	d	E	f	G	h

Fig. D.

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In Fig. D we have in the thick-faced numbers the numerical series 1, 2, 3 . . . . 64 arranged in the sixty-four cells of an eight-square so as to give Caïssa eighty summable paths. By summable we mean giving a constant summation. In this Caïssan square there are, as in an ordinary magic square of the same root, sixteen rook-paths, but there are also sixteen bishop-paths (of which the diagonals of an ordinary magic square are particular cases), thirty-two knight-paths, and at least sixteen special paths.

As an example of a knight-path we give:

40 + 43 + 49 + 62 + 32 + 19 + 9 + 6 = 260,

and of a special path:

40 + 20 + 51 + 2 + 25 + 45 + 14 + 63 = 260.

Each of these paths has seven similar ones parallel to it. Another special path is five cells to the right and one up; *e.g.*:

40 + 23 + 54 + 5 + 25 + 42 + 11 + 60 = 260.

This also has seven similar paths parallel to it; but we have not included these extra-special paths in our number of summable paths.

Any rook-path may be moved *en bloc* from the north side of the square to the south side and *vice versâ*, from the east side to the west side and *vice versâ*, without impairing the square; and the process may be continued indefinitely without hampering the movements or detracting from the remarkable powers of Caïssa.

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Fig. Da.

							61	1	58	3	60	8	63	6	61
						11	52	16	55	14	53	9	50	11	
					47	22	45	17	42	19	44	24	47		
				25	34	27	36	32	39	30	37	25			
			4	64	7	62	5	57	2	59	4				
		54	13	49	10	51	12	56	15	54					
	18	43	20	48	23	46	21	41	18						
40	31	38	29	33	26	35	28	40							

The figure Da shows all the N.E. Bishop-paths, all the E. Ro
aths, and two complete N. Rook-paths. It also shows in the c
ells a complete E.N.E. Knight-path, and, commencing from "
nother may be found to the right of it. Also the figure shows
ishop's *echelon* moved from W. to E., or left to right.

If the bishop-paths, knight-paths, &c., are set out in *echelon* a
ig. Da, similar translations may be made, an *echelon* being ta
n bloc from side to side without disturbing the essence of
rangement.

The thin-faced marginal figures in Fig. D are intended to aid
re in the summations; the letters to afford an index to
nstruction. This we will now give.

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Write the letters *a* to *d* forwards, and *e* to *h* backwards, and th
columns under them, the first half of the series 1 to 64, viz., 1
, forwards and backwards in alternate rows, beginning with
west number as below :

a.	b.	c.	d.	h.	g.	f.	e.
1	2	3	4	5	6	7	[illegible]
16	15	14	13	12	11	10	[illegible]
17	18	19	20	21	22	23	24
32	31	30	29	28	27	26	25

Next, write the letters E to H forwards, the letters A to D ba
ards, and the remainder of the series, viz., 33 to 64, alterna
ckwards and forwards, beginning with the highest number
low :—

E.	F.	G.	H.	D.	C.	B.	A.
64	63	62	61	60	59	58	[illegible]
49	50	51	52	53	54	55	[illegible]
48	47	46	45	44	43	42	[illegible]
33	34	35	36	37	38	39	[illegible]

These columns (or rather, in reference to the eight-square, h
lumns) may now be placed as they stand, to form the Caïssan m
square. Fig. E, and the letters in margin of Fig. D show how this is t accomplished. In Fig E the letters st for the half-columns over which they written. Thus *a* stands for 1, 16, 17, over which it is written ; A. stands for 56, 41, 40, and so on. In our next art we shall give the exploits of Caïss other fields.

a	B	c	D	e	F	g	H
A	b	C	d	E	f	G	h

Fig. E.

(To be continued.)

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CAÏSSAN MAGIC SQUARES.

BY "URSUS."

(Continued from The Queen, p. 142).

PERFECT CAÏSSAN SQUARES are those in which every rook's path, every bishop's path, and every knight's path gives a constant summation. They are, however, pluperfect, having supplementary paths giving the same summation.

To utilise existing terms we shall, for brevity, call any regular path giving the required summation a "magic" path.

It cannot be too distinctly enunciated that any given chequered board must be assumed to have about it eight or more contiguous boards exactly similar to itself, so that when a magic path in the given board comes to or passes the border, it may be continued and completed in the squares (cells) of one or more of the similar contiguous boards. In our former article (see *The Queen*, p. 142) we gave as an example of the perfect Caïssan square, one containing sixty-four cells—in other words, one whose root is 8. A perfect Caïssan square cannot be made with a less number of cells than sixty-four; also some with a greater number fail to give all the Caïssan paths, as we shall now proceed to explain.

The most convenient way of naming a magic square is by the number of cells forming its side. This number, as we have already said, is called the root of the square. Thus the square containing sixty-four cells is an eight-root-square, or, briefly, an eight-square; one containing a hundred cells is a "ten-root-square," or, briefly, a "ten-square."

Roots are either prime or composite; 2, 3, 5, 7, 11, &c., are prime numbers. Of prime numbers, 2 is the only even one, all others being odd. Composite numbers are such as 4, 6, 8, 9, 12, and so on. These may be subdivided thus:

ODD COMPOSITES, *e.g.*, 9, 15, 21, &c.

ODDLY-EVEN COMPOSITES, *e.g.*, 6, 10, 14, &c.

EVENLY-EVEN COMPOSITES, *e.g.*, 4, 8, 12, &c.

There are four corresponding classes of Caïssan squares.

PRIME ROOT CAÏSSAN SQUARES.

These may be easily constructed by general methods. As an illustration, we give in Fig. F the seven-square, but all prime root

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49	31	20	2	40	22	11	49	31
19	1	39	28	10	48	30	19	1
38	27	9	47	29	18	7	38	27
8	46	35	17	6	37	26	8	46
34	16	5	36	25	14	45	34	16
4	42	24	13	44	33	15	4	42
23	12	43	32	21	3	41	23	12
49	31	20	2	40	22	11	49	31
19	1	39	28	10	48	30	19	1

Fig. F.

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Caïssan squares may be formed in the same or a very similar manner.

The numerical series, 1, 2, 3, to 49, is divided into seven sub-series, viz., 1 to 7, 8 to 14, 15 to 21, 22 to 28, 29 to 35, 36 to 42, and 43 to 49.

Commencing with 1 from the north-west corner of the board, we follow the east-nor'-east knight-path, so that 2 falls at the base of the third column from the left on the board above. This in actual practice brings 2 in the given board at the base of the third column from the left. We then continue the knight-path, and on reaching 7, the end of the first sub-series, make a bishop's move, one square (cell) to the south-east for 8, and continue an east-nor'-east knight-path as before. On reaching 14, the end of the second sub-series, another south-east bishop's move of one square (cell) is made, and so we go on till all the forty-nine cells are filled. Of course, whenever, as in the case of 2 above, a knight's leap or bishop's move causes a term of the numerical series to fall outside the margin of the board, on to a cell of one of the similar contiguous boards, the term so falling must be transferred to its *corresponding cell within the margin*. Observe the terms 20, 11, and 4, for example, in Fig. G below. They would be transferred within the border to the cells indicated by the corresponding Roman numerals.

The Caïssan seven-square is perfect in all but certain of the knight-paths.

Fig. G shows the knight-paths which pass through the centre.

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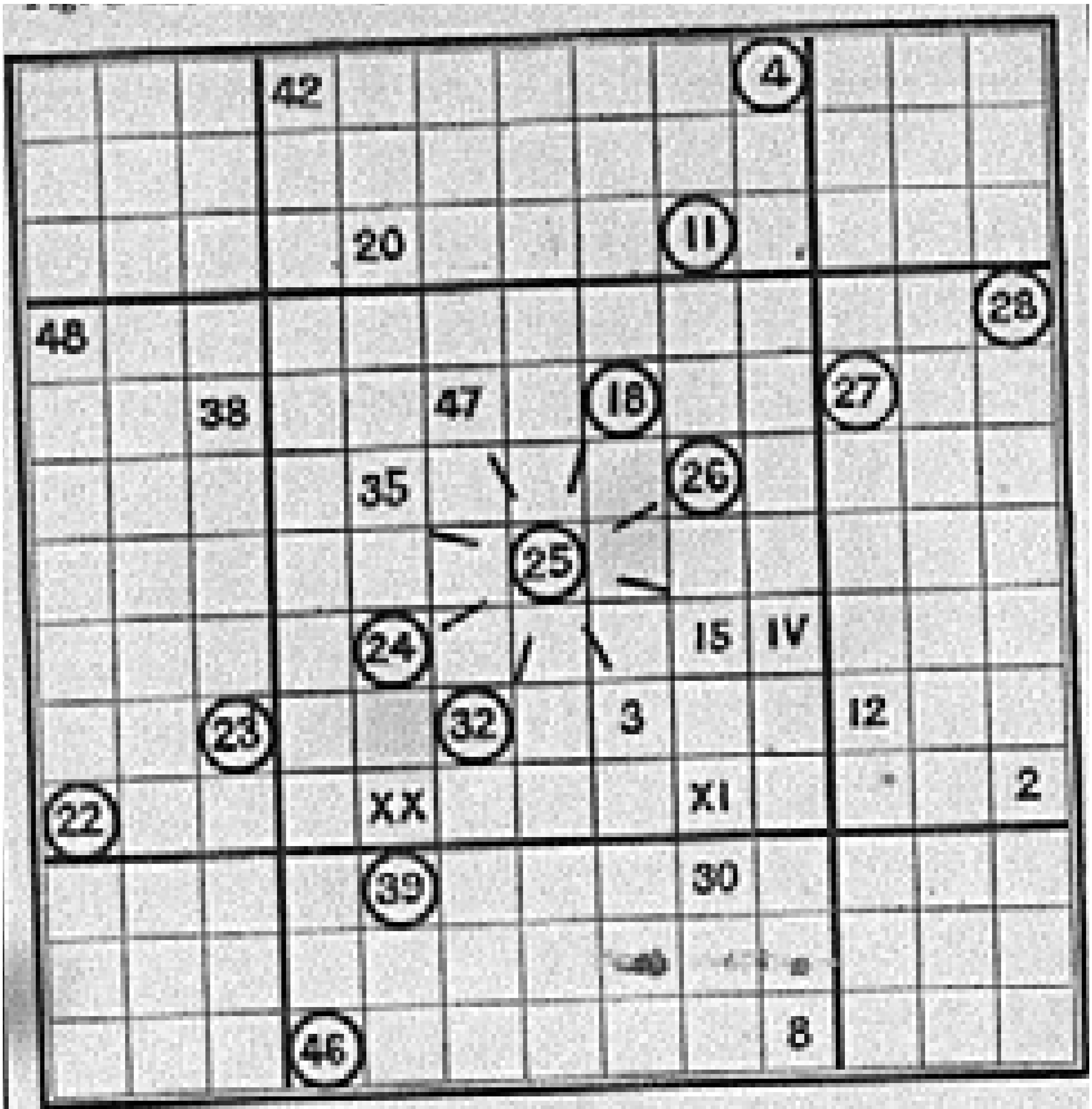

Fig. G.

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They are all "magic" but the paths, which are, so to speak, parallel with the dark-ringed paths, that is to say the N.N.E. and E.N.E. paths are not magic. There are, then, in all, fourteen magic rook-paths, fourteen magic bishop-paths, and sixteen magic knight-paths in the square, each path summing 175. The central or mean term of the series, viz., 25, occupies the middle cell, and any two terms on opposite sides of it, equi-distant from it, and in the same straight line with it, are complementary, summing twice the mean—*i.e.*, 50. Note this in the knight-paths.

Now let us write the series 1—49 in the septenary scale, beginning with 0. The non-mathematical reader can easily do it, if it be remembered that no digit greater than 6 may be employed. The series then becomes 0, 1, &c., to 66. We give a portion of the series in line; but the whole of it will be found in the square, Fig. H.

0, 1......5, 6, 10, 11......15, 16, 20, 21......25, 26, 30, 31......66.

The arrangement shown in Fig. H may easily be obtained from Fig. F by subtracting unity from every term, dividing it by 7, and placing quotient and remainder side by side, *e.g.* (7—1) ÷ 7 = 06. (20—1) ÷ 7 = 25.

The thick-faced left-hand digit is a multiple of the "radix" 7, just as in the common system of notation the left hand of the two digits represents so many tens. We shall call these thick digits radices.

It will now be seen that either the radices by themselves or the units by themselves form a magic square with the series 0 to 6 repeated seven times; also, that the unit-square is an exact counterpart of the radix-square turned as it were about its S.E. diagonal. If Caissa be placed on any one of the forty-nine cells, at least ten magic-paths are open to her either in the radix-square or the unit-square. In other words, in at least ten ways she can pass through

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0_0	5_3	3_6	1_2	6_5	4_1	2_4
3_5	1_1	6_4	4_0	2_3	0_6	5_2
6_3	4_6	2_2	0_5	5_1	3_4	1_0
2_1	0_4	5_0	3_3	1_6	6_2	4_5
5_6	3_2	1_5	6_1	4_4	2_0	0_3
1_4	6_0	4_3	2_6	0_2	5_5	3_1
4_2	2_5	0_1	5_4	3_0	1_3	6_6

Fig. H.

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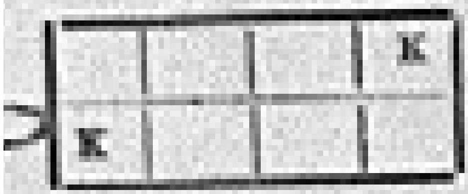
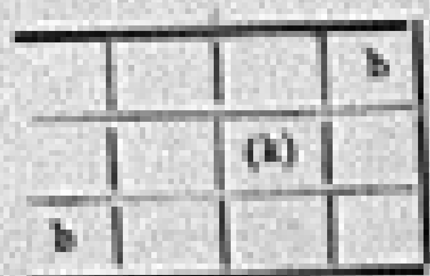
square. In other words, in at least ten ways one can place the digits 0, 1, 2, 3, 4, 5, 6. Hence there are at least seventy magic-paths summing 21, or, if 1 to 7 be taken, summing 28. Twenty-one of the paths are extra Caïssan paths of the form given in the diagram, K K. There are some others which it would be tedious to specify.



The possible number of magic paths from any cell of a prime-root square, when a continuous arithmetical series is employed, is always one less than the root; hence in the seven-square we get (as a rule) only six magic-paths from every starting point. The next prime-root-square, the eleven-square, having ten normal paths, can be made perfectly Caïssan. But the path of construction must not be a knight path. We shall call the construction-path, employed in Fig. J, an oblong bishop's path; it is shown by bb in the annexed diagram. It is compounded of a knight's leap, plus a one-square bishop's move. Starting from 1, we continue the oblong-bishop-path to the end of the first sub-series, i.e. 11, and then make a one-square bishop's move to the S.E., as in forming the seven-square. So, from 12 we continue the oblong-bishop-path to 22, the end of the second sub-series, make a S.E. bishop-move as before, and continue the process till the square is filled. As a check it will be well to complete the S.E. diagonal first. The terms in it will be 1, 13, 25, 37 and so on—a regular series increasing by 12. So from any term already placed we may descend diagonally to the south-east, increasing by 12, unless the term be the end of a sub-series, when we increase by 1 only, thus 6, 18, 30, 42, 54 [66], 67.



The next square, Fig. J, has at least one-hundred-and-ten magic paths, viz., twenty-two rook-paths, twenty-two bishop-paths, forty-four knight-paths, and twenty-two extra paths. It can be reduced to the radix 11, by either of the methods given under the seven-square, but we shall require a new digit for ten. Many mathematicians employ t; we find the Roman numeral X the most

1	83	44	115	76	[illegible]	108	58	19	90	51
63	13	95	45	6	88	38	120	70	31	102
114	75	25	107	57	18	89	50	11	82	43
55	5	87	37	119	69	30	101	62	12	94
106	56	17	99	49	10	81	42	113	74	24
36	118	68	29	100	61	22	93	54	4	86
98	48	9	80	41	112	73	23	105	66	16
28	110	60	21	92	53	3	85	35	117	67
79	40	111	72	33	104	65	15	97	47	8
20	91	52	2	84	34	116	77	27	109	59
71	32	103	64	14	96	46	7	78	39	121

Fig. J.

convenient. The series will now be 1 to 100, or Oo to Xx. (Observe, "100" is now eleven times eleven, or 121 in the ordinary notation.)

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tion.)

In Fig. K we give an eleven-square filled with a series having the radix eleven. Here, again, we have two interwoven Caïssan

00	75	3X	X4	69	23	98	52	17	81	46
57	11	86	40	05	7X	34	X9	63	28	92
X3	68	22	97	51	16	80	45	0X	74	39
4X	04	79	33	X8	62	27	91	56	10	85
96	50	15	8X	44	09	73	38	X2	67	21
32	X7	61	26	90	55	1X	84	49	03	78
89	43	08	72	37	X1	66	20	95	5X	14
25	9X	54	19	83	48	02	77	31	X6	60
71	36	X0	65	2X	94	59	13	88	42	07
18	82	47	01	76	30	X5	6X	24	99	53
64	29	93	58	12	87	41	06	70	35	XX

Fig. K.

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repeated series squares, but now each is perfect. By employing this square first, the corresponding continuous-series square may be easily made.

In the west column the radices form a series increasing by 5 only, we must subtract 11 when getting beyond that number, e.g., 0, 5, 10, 15 (but 15—11 = 4), 9, 14 (i.e., 3), and so on. Having fixed these, the radices in a south-east direction run consecutively.

When the radix square is finished the unit square is easily made by repeating horizontally the arrangement which is perpendicular in the radix-square. Thus, as the west column of radices is **0, 5, X, 4, 9, 3, 8, 2, 7, 1, 6**, so the most northern unit row is 0, 5, X, 4, 9, 3, 8, 2, 7, 1, 6.

Caïssan Evenly-even Squares.

In logical order, squares with odd composite roots should be mentioned next; but these, being the most difficult, are reserved to the last.

The eight-square given in *The Queen*, p. 142, shows the general method of constructing evenly-even squares; and if the eight 1 to 64 or 0 to 63 be reduced to the radix 8, those who wish it can easily perceive the principle of construction.

For simplicity we take in Fig. I. the series 0 to 7. Now 0 + 7 = 1 + 6 = 2 + 5 = 3 + 4. Hence any two of these pairs are equal to any other two; also any pair multiplied by 4, equals the sum of the whole series.

In the eleven-square with radix eleven, it will be seen that in every magic path we have, both in the radices and in the units, every term of the series 0 to X. In the eight-square with radix eight, however, to take the radices, we have in the horizontal rows the complementary pairs repeated, and in the perpendicular rows the whole series. Conversely with the units, we have in the horizontal rows every term of the series 0—7, and in the perpendicular cells give repeated complementary pairs. Any four contiguous cells give, either in radices or units, and therefore in both combined, a constant sum. This suggests divisibility, and, with some modification and a slight sacrifice of path-giving capacity, these evenly-even root squares can be made frangible to a very high degree, which breaks

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``Caïssan magic squares'' by ``Ursus'' (part 2/3, page 11/16)

0_0	7_1	0_2	7_3	0_7	7_6	0_5	7_4
1_7	6_6	1_5	6_4	1_0	6_1	1_2	6_3
2_0	5_1	2_2	5_3	2_7	5_6	2_5	5_4
3_7	4_6	3_5	4_4	3_0	4_1	3_2	4_3
7_0	0_1	7_2	0_3	7_7	0_6	7_5	0_4
6_7	1_6	6_5	1_4	6_0	1_1	6_2	1_3
5_0	2_1	5_2	2_3	5_7	2_6	5_5	2_4
4_7	3_6	4_5	3_4	4_0	3_1	4_2	3_3

Fig. L.

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``Caïssan magic squares'' by ``Ursus'' (part 2/3, page 12/16)

and a slight sacrifice of [illegible] root squares can be made frangible to a very high degree.

In Fig. M we have a Caïssan sixteen-square, which breaks up into four perfect Caïssan eight-squares. These eight-squares we shall, for distinction, call "squarelets." The two eastern may change places with the two western, the two northern may change places with the two southern, the N.E. with the S.W., the N.W. with the S.E., without altering or impairing the properties of the whole square. Also, each of the squarelets may be turned through one, two, or three right angles (so long as its companions are treated in a precisely similar way) without injuring the great square. Again, if the squarelets be split into halves vertically, any right-half will, with any left-half, make a perfect Caïssan eight-square. The only essential point in which this differs from the ordinary Caïssan sixteen-square is that, in following a knight's path, eight cells must be taken in one squarelet before passing to another squarelet. The same rule applies in principle to the extra paths.

The method of constructing this square is extremely simple. Having ruled the lines for a sixteen-square, divide it into four squarelets, placing 1, 2, 3, 4 in natural-square order in the corners of the squarelets. These become the index numbers. Fill No. 1 squarelet with the light series, 1, 5, 9, &c., increasing by four, and with the heavy complementary series, 256, 252, 248, &c., decreasing by four in the order given in Figs. L, M, N, &c. When No. 1 squarelet is completed, make every term of the light series in No. 2 squarelet *greater by* 1, and every term of the heavy series *less by* 1, than the corresponding term in No. 1 squarelet. Each term in squarelet No. 3 will, of course, be *greater or less by* 2, each term in No. 4 squarelet *greater or less by* 3, though it is easier to construct No. 4 from No. 3.

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``Caïssan magic squares'' by ``Ursus'' (part 2/3, page 13/16)

with the ...

1	232	9	240	29	252	21	244	2	231	10	239	30	251	22	243
61	220	53	212	33	200	41	208	62	219	54	211	34	199	42	207
65	168	73	176	93	188	85	180	66	167	74	175	94	187	86	179
125	156	117	148	97	136	105	144	126	155	118	147	98	135	106	143
228	5	236	13	256	25	248	17	227	6	235	14	255	26	247	18
224	57	216	49	196	37	204	45	223	58	215	50	195	38	203	46
164	69	172	77	192	89	184	81	163	70	171	78	191	90	183	82
160	121	152	113	132	101	140	109	159	122	151	114	131	102	139	110
3	230	11	238	31	250	23	242	4	229	12	237	32	249	24	241
63	218	55	210	35	198	43	206	64	217	56	209	36	197	44	205
67	166	75	174	95	186	87	178	68	165	76	173	96	185	88	177
127	154	119	146	99	134	107	142	128	153	120	145	100	133	108	141
226	7	234	15	254	27	246	19	225	8	233	16	253	28	245	20
222	59	214	51	194	39	202	47	221	60	213	52	193	40	201	48
162	71	170	79	190	91	182	83	161	72	169	80	189	92	181	84
158	123	150	115	130	103	138	111	157	124	149	116	129	104	137	112

Fig. M.

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``Caïssan magic squares'' by ``Ursus'' (part 2/3, page 14/16)

1	224	49	240	2	223	50	239	3	222	51	238	4	221	52	237
113	176	65	160	114	175	66	159	115	174	67	158	116	173	68	157
208	17	256	33	207	18	255	34	206	19	254	35	205	20	253	36
192	97	144	81	191	98	143	82	190	99	142	83	189	100	141	84
5	220	53	236	6	219	54	235	7	218	55	234	8	217	56	233
117	172	69	156	118	171	70	155	119	170	71	154	120	169	72	153
204	21	252	37	203	22	251	38	202	23	250	39	201	24	249	40
188	101	140	85	187	102	139	86	186	103	138	87	185	104	137	88
9	216	57	232	10	215	58	231	11	214	59	230	12	213	60	229
121	168	73	152	122	167	74	151	123	166	75	150	124	165	76	149
200	25	248	41	199	26	247	42	198	27	246	43	197	28	245	44
184	105	136	89	183	106	135	90	182	107	134	91	181	108	133	92
13	212	61	228	14	211	62	227	15	210	63	226	16	209	64	225
125	164	77	148	126	163	78	147	127	162	79	146	128	161	80	145
196	29	244	45	195	30	243	46	194	31	242	47	193	32	241	48
180	109	132	93	179	110	131	94	178	111	130	95	177	112	129	96

Fig. N.

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easier to construct No. [illegible] from No. [illegible]

By sacrificing the knight-paths we can get a still greater amount of frangibility. In Fig. N we have the "Caïssan Square of Squares," breaking up into sixteen squarelets. It has the following properties: All the properties, except the knight-paths of the Caïssan sixteen-square, and all the properties, with that exception of the square given in Fig. M. Besides these, as the squarelets stand, any contiguous four of them will make a Caïssan eight-square, and any contiguous nine will form a Caïssan twelve-square. In fact, twelve-squares and eight-squares may be made in endless variety, by merely noticing the index numbers of the squarelets.

properties: All the properties, except the knight-paths of the Caïssan sixteen-square, and all the properties, with that exception of the square given in Fig. M. Besides these, as the squarelets stand, any contiguous four of them will make a Caïssan eight-square, and any contiguous nine will form a Caïssan twelve-square. In fact, twelve-squares and eight-squares may be made in endless variety, by merely noticing the index numbers of the squarelets.

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[illegible]

Thus, if the indices are—

2	4
6	8

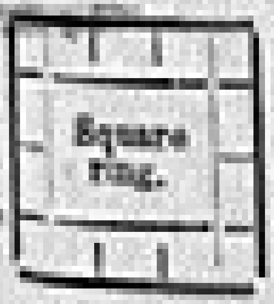
or

3	6
9	12

we shall get Caïssan eight squares, for 2 + 8 = 6 + 4, and 3 + 12 = 9 + 6. So, for example, if the indices are—

1	3	5
6	8	10
11	13	15

we shall have a twelve-square, because 11 + 8 + 5 = 13 + 10 + 1 = 15 + 6 + 3 = 24. The "Caïssan Square of Squares" has, further, the properties of a nagel-square. By "ring" we mean a group of cells forming a hollow square, as in the annexed diagram.



Of course we can strip off the twelve external squarelets, leaving an eight-square; but, besides this, we can strip off the outer ring of cells, and the next but one; then, taking out the numbers, 17, 65, 174, 253, 161, 241, 32, and 80 in the dark cells, *fit the remainder of that ring* to the inner square so as to have a Caïssan twelve-square. The process may be repeated with this twelve-square. This remarkable square of squares has many other singular properties, as any one may ascertain by trying experiments with it. It is constructed on the same principle as that given in Fig. M. Squarelet No. 1 is filled with the light series 1, 17, 33, &c., increasing by 16, and with the heavy complementary series 256, 240, 224, &c., decreasing by 16.

The terms in No. 2 are made each *greater or less* by 1, those in No. 3 greater or less by 2, and so on till all are filled.

(To be continued.)

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CAÏSSAN MAGIC SQUARES.

BY "URSUS."

(Concluded from The Queen of Sept. 10.)

CAÏSSAN SQUARES WITH ODDLY-EVEN ROOTS.

THESE cannot be made absolutely perfect. We may make them with mutilated series, as 1, 2, 3, 4, 5, 7, &c., or imperfectly Caïssan, as in Fig. P.

9 9	0 8	+ 0 2	+ 9 3	+ 0 4	* 1 0	9 2	0 3	9 4	9 0
8 0	1 1	8 2	1 3	8 4	1 8	8 7	1 6	8 5	1 9
7 9	2 8	7 7	2 6	7 5	2 1	7 2	2 3	7 4	2 0
6 0	3 1	6 2	3 3	6 4	3 8	6 7	3 6	6 5	3 9
5 9	4 8	5 7	4 6	5 5	4 1	5 2	4 3	5 4	4 0
* 1	8 1	1 2	8 3	1 4	8 8	1 7	8 6	1 5	9 8
2 9	7 8	2 7	7 6	2 5	7 1	2 2	7 3	2 4	7 0
3 0	6 1	3 2	6 9	3 4	6 8	3 7	6 6	3 5	6 9
4 9	5 8	4 7	5 6	4 5	5 1	4 2	5 3	4 4	5 0
0 9	9 1	9 7	0 6	9 5	8 9	0 7	9 6	0 5	0 0

FIG. P.

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Here we have a perfect magic square having rook-paths and diagonals with constant summations, and on removing the outer ring it becomes a perfect Caïssan square.

To construct it, take the series 1 to 100, or, to show more clearly the principle of formation, the series 0 to 99. Throw out for the ring 0 to 9, 90 to 99 and all the terms having either 9 or 0 in the unit's place. In a "natural square" these fall in the outer ring. Fill the centre eight-square with the remaining terms 11 to 48 for the light series, 88 to 51 for the heavy series, as in the preceding squares. At the corners of the ring place 0, 9, 90, and 99 in inverted natural square order. In the north and in the west cell-rows place the terms which would be there if the law evident in the central board prevailed throughout. Place opposite each of these terms, on the further side of the ring (*i.e.*, in the south and east cell rows), its complementary term. Then transpose 1 and 10, the terms marked by the asterisk, and instead of the terms 97, 6, and 95 place 2, 93, 4, their complementary terms as shown by the crosses. The square will then be complete as in Fig. P.

CAÏSSAN SQUARES WITH ODD COMPOSITE ROOTS.

These require special constructions, but yield a large number of magic paths. Thus the fifteen-square has at least three hundred and twenty-four distinct magic paths. It is given in Fig. Q:

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129	6	207	131	2	204	126	12	206	122	9	201	132	11	197
76	193	68	78	195	61	88	188	63	90	181	73	83	183	75
179	110	49	175	112	59	170	109	55	172	119	50	169	115	52
159	36	147	161	32	144	156	42	146	152	39	141	162	41	137
16	223	98	18	225	91	28	218	93	30	211	103	23	213	105
134	5	199	130	7	209	125	4	205	127	14	200	124	10	202
84	186	72	86	182	69	81	192	71	77	189	66	87	191	62
166	118	53	168	120	46	178	113	48	180	106	58	173	108	60
164	35	139	160	37	149	155	34	145	157	44	140	154	40	142
24	216	102	26	212	99	21	222	101	17	219	96	27	221	92
121	13	203	123	15	196	133	8	198	135	1	208	128	3	210
89	185	64	85	187	74	80	184	70	82	194	65	79	190	67
174	111	57	176	107	54	171	117	56	167	114	51	177	116	47
151	43	143	153	45	136	163	38	138	165	31	148	158	33	150
29	215	94	25	217	104	20	214	100	22	224	95	19	220	97

Fig. Q.

And in Fig. R we show the extra Caïssan paths, the ordinary ones being taken for granted. If Caïssa be placed on the cell marked A, she can move in the direction A B, A b, A C, A c, and so on to

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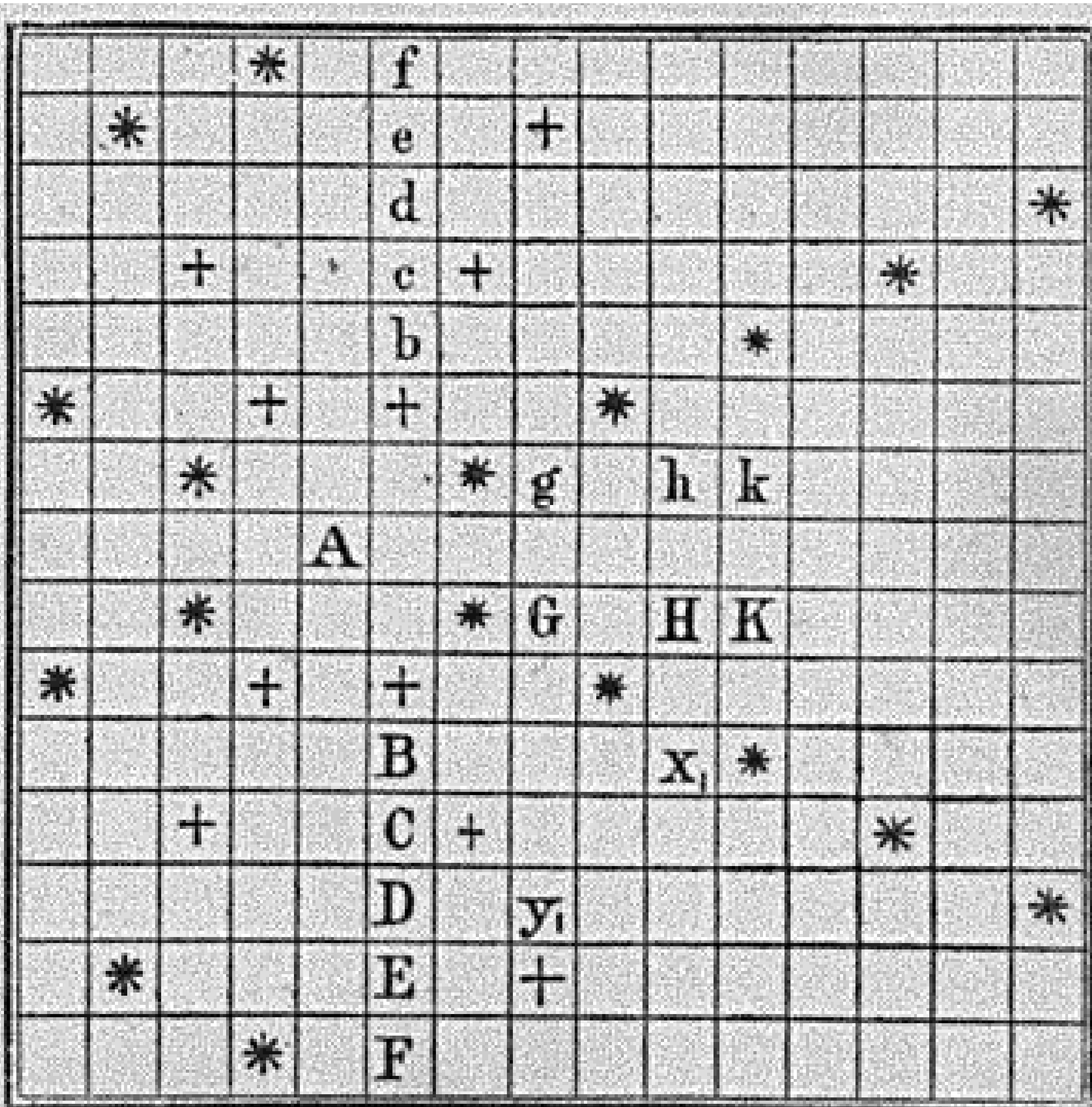

FIG. R.

make magic paths. Also, if A happens to be the central cell, the construction paths Ax, Ay, and the knight's path shown by the asterisks, will also be magic.

The construction is not so difficult as might be imagined :

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The construction is not so difficult as might be imagined:

Aa	b	c	d	e
d	Ee	a	b	c
b	c	Dd	e	a
e	a	b	Cc	d
c	d	e	a	Bb

It will have been noticed that in a prime root square it is very easy to make each path contain every term of a repeated series; and if we have two series interwoven, then each path must contain every term of both one series and the other. Hence, if we have a series representing the multiples of the radix, and another series representing the units less than the radix, each path is bound to give us the sum required. The most apparent cases are those of the S.E. diagonals of the seven-square and the eleven-square, where we have 0_0, 1_1, 2_2, 3_3, &c., just as in the accompanying diagram we have Aa, Ee, Dd, &c. The order of the terms is quite immaterial, so long as the same order is maintained throughout. In composite root-squares, however, we have certain complementary cell-groups repeated, as 0, 7; 1, 6; 2, 5; 3, 4; in the eight-square. Fig. S., illustrating the principle of the fifteen-square, will furnish another example.

Hh	Oe	Mk	Hj	Oa	Mh	He	Ok	Mj	Ha	Oh	Me	Hk	Oj	Ma
Eo	Ll	Dg	Eb	Ln	Do	El	Lg	Db	En	Lo	Dl	Eg	Lb	Dn
Km	Gd	Cc	Ki	Gf	Cm	Kd	Gc	Ci	Kf	Gm	Cd	Kc	Gi	Cf
Jh	Be	Ik	Jj	Ba	Ih	Je	Bk	Ij	Ja	Bh	Ie	Jk	Bj	Ia
Ao	Nl	Fg	Ab	Nn	Fo	Al	Ng	Fb	An	No	Fl	Ag	Nb	Fn
Hm	Od	Mc	Hi	Of	Mm	Hd	Oc	Mi	Hf	Om	Md	Hc	Oi	Mf
Eh	Le	Dk	Ej	La	Dh	Ee	Lk	Dj	Ea	Lh	De	Ek	Lj	Da
Ko	Gl	Cg	Kb	Gn	Co	Kl	Gg	Cb	Kn	Go	Cl	Kg	Gb	Cn
Jm	Bd	Ic	Ji	Bf	Im	Jd	Bc	Ii	Jf	Bm	Id	Jc	Bi	If
Ah	Ne	Fk	Aj	Na	Fh	Ae	Nk	Fj	Aa	Nh	Fe	Ak	Nj	Fa
Ho	Ol	Mg	Hb	On	Mo	Hl	Og	Mb	Hn	Oo	Ml	Hg	Ob	Mn
Em	Ld	Dc	Ei	Lf	Dm	Ed	Lc	Di	Ef	Lm	Dd	Ec	Li	Df
Kh	Ge	Ck	Kj	Ga	Ch	Ke	Gk	Cj	Ka	Gh	Ce	Kk	Gj	Ca
Jo	Bl	Ig	Jb	Bn	Io	Jl	Bg	Ib	Jn	Bo	Il	Jg	Bb	In
Am	Nd	Fc	Ai	Nf	Fm	Ad	Nc	Fi	Af	Nm	Fd	Ac	Ni	Ff

Fig. 8.

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Having no digit greater than 9, let the letters A to O consecutively stand for the series O to 14, in the following order; for the units the small letters:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12,	13,	14,	0

and for the radices (strictly multiples of the radix, 15), the capitals:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
15,	30,	45,	60,	75,	90,	105,	120,	135,	150,	165,	180,	195,	210,	0

Now the sum of the series o ...... n, is 15 (o + n) ÷ 2 = 105, but in virtue of being a multiple of 3 (greater than twice 3), it breaks up into triplets, each triplet having a constant sum; also, in virtue of being a multiple of 5 (greater than twice 5), it breaks up into five-term groups also having a constant sum. Hence (omitting the plus signs for the sake of compactness):

hom = eld = kgc = jbi = anf = 21

and hekja = olgbn = mdcif = 35

So that, to take examples:

5 hom = 3 hekja = o ...... n.

That is, five repeated triplets, or three repeated five-groups, give the same sum as the whole series.

So,	h	e	k	j	a,	i.e.,	8,	5,	11,	10,	1	=	35
	o	l	g	b	n		0,	12,	7,	2,	14	=	35
	m	d	c	i	f		13,	4,	3,	9,	6	=	35
							21,	21,	21,	21,	21	=	105

forms a parallelogram, each of the long rows summing one-third of the whole series, and each of the short rows one-fifth of the series.

For Girtonians and Newnhamites we may state the general rule thus:—If the series consists of nm terms, it will break up into n groups (of m in a group), each summing $\frac{1}{n}$ of the whole series, and also into m groups (of n in a group), each summing $\frac{1}{m}$ of the whole series.

Of course nm must be either odd or evenly-even.

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This reasoning applies equally to the radix-series, for the sum of that is merely fifteen times the sum of the unit series. Now, let S, s stand for the sum of the whole series, T, t for the sum of a triplet, and V, v for the sum of a five-group, the capitals, of course, representing radices. Then, starting from the S.W. corner, we have the N. rook path summing:

$$3\,V + 5\,t = 1575 + 105 = 1680$$

the E. rook path:

$$5\,T + 3\,v = 1575 + 105 = 1680$$

a N.E. bishop path:

$$S \quad + \quad s = 1575 + 105 = 1680$$

an E.N.E. knight-path, indicated by the dark cells:

$$S \quad + \quad s = 1575 + 105 = 1680$$

and so on with the other paths. In the numbered square, Fig. Q, we have added unity to each cell, so as to make the series 1-15, and the sum $S + s = 1695$.

For a twenty-one square we should require triplets and seven-groups; the object of this double system of grouping is the prevention of a recurrence of any particular combination, *e.g.*, in our figure the combination A m occurs in the S.W. corner only.

It may now be seen why oddly-even roots fail to give Caïssan squares. Take as an example the root 6. The series 1—6 should, the sum being a multiple of 3, break into triplets; but this it will not do. In common magic squares the excess of one triplet may, by trial, be made to balance the defect of another, but even in these an oddly-even square in which there is a regular law is a mathematical impossibility.

The idea of the foregoing Caïssan magic squares was suggested partly by the Brahminical squares, that from time immemorial have been used by the Hindoos as talismans; partly by an article by the Rev. A. H. Frost, M.A., of St. John's College, Camb., on "Nasik Squares," in No.57 of the "Quarterly Journal of Pure and Applied Mathematics" (1877). Our squares are, however, mostly original, as are the methods of construction, though one or two, notably that for the fifteen-square, may by Girtonians and Newnhamites readily be translated into the (mathematically) purer language adopted in Mr Frost's able paper. The epithet "Caïssan" describes the distinguishing characteristics of the squares, as the paths include all possible continuous chess moves. We commenced with Caïssa's

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mythical history; let us conclude with a few facts less apocryphal. We have alluded to the Brahminical squares. Of these the favorites are the four-square and the eight-square. We have shown that the latter is, from our point of view, the first perfect one: in fact, as the Policeman of Penzance would say, "taking one consideration with another," it may be pronounced the best of the lot. Now, *chaturanga*, the great-grandfather of chess, is of unknown antiquity, and Duncan Forbes gives translations of venerable Sanskrit manuscripts, which describe the pieces, moves, and mode of playing the game. *Chatur anga*, means four arms, *i.e.*, the four parts of an army—elephants, camels, boats (for fighting on the rivers, Sanskrit *roka*, a boat; hence rook), and foot-soldiers, the whole commanded by the kaisar or king. At first there were four armies, two as allies against two others; afterwards the allies united, one of the kings becoming prime minister with almost unlimited powers, and in the chivalrous west being denominated Queen. We have already noticed that the Russians, who are inveterate chess-players, and who probably got their national game direct from the fountain head, endow their queen with the powers of all the other pieces. So then in the west we have king, queen, bishop (ex-elephant), knight (ex-camel), castle or rook, and pawn, the line soldier on whom, after all, military success depends. Sir William Jones, who wrote "Caïssa" when he was a boy, thinking in Greek, but then little versed in Sanskrit, turned the corrupt Saxon "chess" into the pseudo-classic Caïssa. Strange coincidence, for much later, his admirer, Duncan Forbes, traces "chess," by such links as the French *echecs*, English *check* and *check mate*, German *schach matt*, to the Persian *Shah mat*; Shah being their rendering of the Sanskrit or Aryan *Kaisar*, still retained by Germans, and by Russians in Czar.

Sir William's Caïssa would pass very well for the feminine of Kaisar; but this is not the coincidence we wish to accentuate.

Both "chess" and "Caïssan" squares, under whatever name the reader pleases, have been known in India—the nursery of civilisation—from the remotest antiquity; hence there would seem to be a close connection between them. India is now the greatest possession of the British crown, and our Queen, as everyone knows, bears the masculine title of Kaisar-in-Hind.

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``Caïssan magic squares'' by ``Ursus'' (part 1/3)

THE BOUDOIR.

TO CORRESPONDENTS.

WOMEN AT THE UNIVERSITY OF LONDON.

NOTES AND QUERIES.

PASTIMES.

RULES FOR "QUEEN" DOUBLE ACROSTIC PRIZES.

TO CORRESPONDENTS.

DOUBLE ACROSTIC No. 718.

Why is not former latter cheap?
Echo does not answer why;
We've to pay, while dealers keep
Up a huge monopoly.

1. If you would have this quality of floor,
With silicate of soda brush it o'er.
2. "Even the pious priest,
As he implored his God to strengthen us,
He groaned and cursed."
3. He was well blooded every day,
Warm water had for drink;
But, to the doctor's great dismay,
The patient 'gan to sink.
4. "Exuberant district, whose much-favoured plains
Boast their four harvests in the circling year." CRACK.

SOLUTION OF DOUBLE ACROSTIC No. 716.

H	HEAT-WAVES	S
E	EU	U
A	ANNEXATION	N
T	TUNIS	S
W	WONT	T

CAISSAN MAGIC SQUARES.

By "URSUS."

ONCE UPON A TIME, when Orpheus was a little boy, long before the world was blessed with the "Eastern Question," there dwelt in the Balkan forests a charming nymph by name Caïssa. The sweet Caïssa roamed from tree to tree in Dryad meditation fancy free. As for trees, she was, no doubt, most partial to the box and the ebony. Mars read of her charms in a "society" paper, saw her photograph, and started by the first express for Thracia. He came, he saw, but he conquered not—in fact, was ingloriously repulsed. As he was wandering by the Danube blurting out, to give vent to his chagrin, many unparliamentary expressions, a Naiad advised him to go to Euphron, who ruled over a sort of Lowtherian Arcadia. Euphron good-naturedly produced from his stores a mimic war game, warranted to combine amusement with instruction. The warrior—in mufti—took the game to Caïssa, taught her how to play, called it after her name, and, throwing off his disguise, proposed. The delighted Caïssa consented to become Mrs Mars. He spent the honeymoon in felling box trees and ebony ditto, she in fashioning the wood to make tesselated boards and quaint chessmen to send all over the uncivilised world.

Such is our profane version of the story, told so prettily in Ovidian verse by Sir William Jones, the Orientalist. Such is Caïssa, as devoutly believed in by Phillidorians as though she were in Lempriere.

Our Caïssa, however, shall herself be a chessman—we beg her pardon—chesswoman. Like the "queen" in the Russian game, she shall have the power of every man, moving as a king, rook, bishop, or pawn (like the queen in our game), and also as a knight.

Caïssa then is complete mistress of the martial board. In Fig. A¹, suppose her to be at R; she can as a rook move in the direction r¹, r², r³, or r⁴. If at B in Fig. A², she can move, as a bishop, in

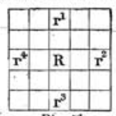

Fig. A¹

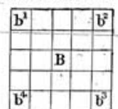

Fig A²

the direction b¹, b², b³, or b⁴. In Fig. B¹, suppose her to be at K; she can, as a knight, leap to k¹, k², k³, k⁴, k⁵, k⁶, k⁷, or k⁸.

These sixteen moves may conveniently be indicated by the points

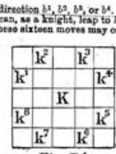

Fig. B¹

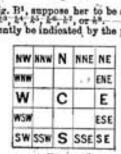

Fig. B²

of the compass as in Fig. B². The rook moves E, W, N, or S. The bishop, NE, NW, SE, or SW. The knight, WNW, NNW, NNE, ENE, ESE, SSE, SSW, or WSW. Caïssa can go in any one of these directions.

We must now describe what is meant by a *path*. A magic square is most conveniently named by the number of cells in one of its sides. Hence the ordinary chessboard of sixty-four cells is an eight-square; a square with eighty-one cells is a nine-square, and so on. A path in an eight-square consists of eight cells; a path in a nine-square consists of nine cells, and so on; to speak generally, a path contains as many cells as the side of the square.

When Caïssa is following a path, she must keep on moving or leaping in the same direction to complete the required number of cells. Thus, if she starts on a north rook-path she must complete the required number of cells in that path; so, if she commences an east-nor'-east knight-path, she must leap the required number of cells in that path.

Lastly, and this is the only difficulty, it must be assumed that the chess board or eight-square on which Caïssa starts has chessboards or eight-squares similar to itself contiguous on every side. This is partly shown in Fig. C. The thick line surrounding 64 small

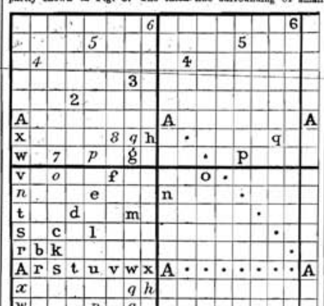

Fig. C.

squares (called cells when speaking of magic squares), in the lower left-hand or SW. portion of the figure, represents, say, a chessboard. The similar figures above and to the right represent three imaginary chessboards contiguous to the first, and the one column outside the thick line to the extreme right represents a portion of other chessboards, the remainder not being shown for want of space. There might also be imaginary boards to the left and below, but they are not necessary for our present illustration of Caïssa's moves.

Assume that Caïssa is placed in the cell marked A in the lower left-hand board, which, for distinction, we shall call the *home board*, and that she starts on her paths from there. A*r* to *x* is a rook-path; A*b* to *h*, a bishop-path; A*k* to *q*, a knight-path. Other rook and bishop paths are indicated by dots in the imaginary boards; and a path of another kind, called Caïssa's special path, is

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shown by the numerals in the upper boards, starting from A in the upper left-hand board (which stands for 1) and going to 2, 3, 4, 5, 6. The completion of this path above and beyond 6 is omitted for want of space. Caïssa has at least two special paths in a perfect Caïssan magic square.

Now, calling A the *initial cell*, it will be observed that when Caïssa has completed an eight-cell path on the *home board*, a one-cell move brings her to the initial cell of one of the imaginary contiguous boards. When, however, she cannot complete such a path on the home board by continuing in the direction in which she started, she must complete it in those cells of the *home board* which are *similarly situated* to those in the imaginary boards in which the path, when continued, would fall. So, with a numerical series, the terms falling in the cells of contiguous boards must be transferred to corresponding cells of the *home board*. This is shown by the thin-faced letters in Fig. C.

From the above considerations we may see that a path can be completed by working, so to speak, *backwards*, and that, though Caïssa has, including her special paths, twenty *initial moves* from any given cell, she has only ten paths; for a North-path is identical with a South-path, a North-east with a South-west, and so on. But as there are in an eight-square sixty-four starting points, or eight in any row, she has in that square *eighty distinct paths*, each of which, when the square is properly filled with numbers, gives a constant summation. If the numbers 1 to 64 are chosen, as seems most natural, the constant summation of each path will be 260.

28	a	B	c	D	e	F	g	H
61	1	58	3	60	8	63	6	61
52	16	55	14	53	9	50	11	52
45	17	42	19	44	24	47	22	45
36	32	39	30	37	25	34	27	36
5	57	2	59	4	64	7	62	5
12	56	15	54	13	49	10	51	12
21	44	18	43	20	48	23	46	21
28	40	31	38	29	33	26	35	28
61	A	b	C	d	E	f	G	h

Fig. D.

In Fig. D we have in the thick-faced numbers the [illegible] series 1, 2, 3 . . . . 64 arranged in the sixty-four cells of an eight-square so as to give Caïssa eighty summable paths. By summable we mean giving a constant summation. In this Caïssan square there are, as in an ordinary magic square of the same root, sixteen rook-paths, but there are also sixteen bishop-paths (of which the diagonals of an ordinary magic square are particular cases), thirty-two knight-paths, and at least sixteen special paths.

As an example of a knight-path we give:

$$40 + 43 + 49 + 62 + 32 + 19 + 9 + 6 = 260,$$

and of a special path:

$$40 + 20 + 51 + 2 + 25 + 45 + 14 + 63 = 260.$$

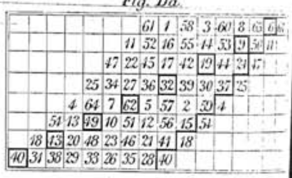
Each of these paths has seven similar ones parallel to it. Another special path is five cells to the right and one up; *e.g.*:

$$40 + 23 + 54 + 5 + 25 + 42 + 11 + 60 = 260.$$

This also has seven similar paths parallel to it; but we have not included these extra-special paths in our number of summable paths.

Any rook-path may be moved *en bloc* from the north side of the square to the south side and *vice versâ*, from the east side to the west side and *vice versâ*, without impairing the square; and the process may be continued indefinitely without hampering the movements or detracting from the remarkable powers of Caïssa.

Fig. Da.



The figure Da shows all the N.E. Bishop-paths, all the E. [illegible] paths, and two complete N. Rook-paths. It also shows in the [illegible] cells a complete E.N.E. Knight-path, and, commencing from "[illegible]" another may be found to the right of it. Also the figure shows [illegible] Bishop's *echelon* moved from W. to E., or left to right.

If the bishop-paths, knight-paths, &c., are set out in *echelon* [illegible] Fig. Da, similar translations may be made, an *echelon* being taken *en bloc* from side to side without disturbing the essence of the arrangement.

The thin-faced marginal figures in Fig. D are intended to aid the eye in the summations; the letters to afford an index to the construction. This we will now give.

Write the letters *a* to *d* forwards, and *e* to *h* backwards, and [illegible] in columns under them, the first half of the series 1 to 64, viz., 1 to 32, forwards and backwards in alternate rows, beginning with the lowest number as below:

a.	b.	c.	d.	h.	g.	f.	e.
1	2	3	4	5	6	7	[illegible]
16	15	14	13	12	11	10	[illegible]
17	18	19	20	21	22	[illegible]	[illegible]
32	31	30	29	28	27	[illegible]	[illegible]

Next, write the letters E to H forwards, the letters A to D backwards, and the remainder of the series, viz., 33 to 64, alternately backwards and forwards, beginning with the highest number [illegible] below:—

E.	F.	G.	H.	D.	C.	B.	A.
64	63	62	61	60	59	58	[illegible]
49	50	51	52	53	54	55	[illegible]
48	47	46	45	44	43	42	[illegible]
33	34	35	36	37	38	39	[illegible]

These columns (or rather, in reference to the eight-square, half-columns) may now be placed as they stand, to form the Caïssan magic square, Fig. E, and the letters in the margin of Fig. D show how this is accomplished. In Fig E the letters stand for the half-columns over which they are written. Thus *a* stands for 1, 16, 17, [illegible] over which it is written; A. stands for [illegible] 56, 41, 40, and so on. In our next [illegible] we shall give the exploits of Caïssa [illegible] other fields.

a	B	c	D	e	F	g	H
A	b	C	d	E	f	G	h

Fig. E.

(To be continued.)

BLINDNESS.—At least one-fourth of those suffering from blindness [illegible] clearly trace their calamity to the use of spectacles imperfectly [illegible]

The Queen, the Lady's Newspaper & Court Chronicle
vol. 70, p. 276--277 (Sepember 10, 1881), reel 20

``Caïssan magic squares'' by ``Ursus'' (part 2/3)

PASTIMES.

RULES FOR "QUEEN" DOUBLE ACROSTIC PRIZES.

1. At the end of the quarter (last Saturday in September) a prize value £1 1s. will be given to the solver of the greatest number of Queen double acrostics during the quarter. [illegible]

[illegible]

TO CORRESPONDENTS.

[illegible]

DOUBLE ACROSTIC No. 723.

My latter, with former, makes passable rhyme;
My latter at former gives excellent reason;
But, strange to relate, it is heard at the time
That's known as the silly or gullible season.
'Tis all about science so deep and so wide,
The theories of molecules, measures, and forces;
To former, their hobbies philosophers ride
As Turpin and others did favourite horses.

1. Legitimate right was the cause he espoused
In battle: but never a reader supposes
That he was, at home, of necessity housed
At former, or grew there his favourite roses.
2. Molluscs: each shelly-one's armed with a lid,
And 'gainst an intruder can readily close it.
3. A badger; though deftly the honey is hid,
He speedily knows it—he'll speedily nose it.
4. He put in good order the planets and sun
And left 'em all tidy and ready for Newton;
The Latinised form of his name let us shun,
For he was a German, a deep and acute 'un.—URSUS.

SOLUTION OF DOUBLE ACROSTIC, No. 721.

S SIZER* S
E ELSEWHERE* E
A ARENA A
T TENANT* T CRACK.

* Alternatives allowed, the lights being rather vague.

[Correct replies have been received from A. J. S., Blue, Chattisgurh, Charon, Calais, Emilie, E. C., J. E. W., Kelt, Katydid, Mrs Mums, O. W. Owl, P. H. E., Pollin, Phantasmagoria, Respice Finem, Sea Serpent, Shenstone, Tut-o-mum, Zebra. All others are wrong, or do not conform to rules.—ED.]

CAÏSSAN MAGIC SQUARES.

BY "URSUS."

(Continued from The Queen, p. 142).

PERFECT CAÏSSAN SQUARES are those in which every rook's path, every bishop's path, and every knight's path gives a constant summation. They are, however, pluperfect, having supplementary paths giving the same summation.

To utilise existing terms we shall, for brevity, call any regular path giving the required summation a "magic" path.

It cannot be too distinctly enunciated that any given chequered board must be assumed to have about it eight or more contiguous boards exactly similar to itself, so that when a magic path in the given board comes to or passes the border, it may be continued and completed in the squares (cells) of one or more of the similar contiguous boards. In our former article (see *The Queen*, p. 142) we gave, as an example of the perfect Caïssan square, one containing sixty-four cells—in other words, one whose root is 8. A perfect Caïssan square cannot be made with a less number of cells than sixty-four; also some with a greater number fail to give all the Caïssan paths, as we shall now proceed to explain.

The most convenient way of naming a magic square is by the number of cells forming its side. This number, as we have already said, is called the root of the square. Thus the square containing sixty-four cells is an eight-root-square, or, briefly, an eight-square; one containing a hundred cells is a "ten-root-square," or, briefly, a "ten-square."

Roots are either *prime* or *composite*; 2, 3, 5, 7, 11, &c., are prime numbers. Of prime numbers, 2 is the only *even* one, all others being *odd*. Composite numbers are such as 4, 6, 8, 9, 12, and so on. These may be subdivided thus:

ODD COMPOSITES, *e.g.*, 9, 15, 21, &c.
ODDLY-EVEN COMPOSITES, *e.g.*, 6, 10, 14, &c.
EVENLY-EVEN COMPOSITES, *e.g.*, 4, 8, 12, &c.

There are four corresponding classes of Caïssan squares.

PRIME ROOT CAÏSSAN SQUARES.

These may be easily constructed by general methods. As an illustration, we give in Fig. F the seven-square, but all prime root

42	31	20	2	40	22	11	49	31
13	1	39	28	10	48	30	19	1
38	27	9	47	29	18	7	38	27
8	46	35	17	6	37	26	8	46
34	16	5	36	25	14	45	34	16
4	42	24	13	44	33	15	4	42
23	12	43	32	21	3	41	23	12
49	31	20	2	40	22	11	49	31
19	1	39	28	10	48	30	19	1

Fig. F.

Caïssan squares may be formed in the same or a very similar manner.

The numerical series, 1, 2, 3, to 49, is divided into seven sub-series, viz., 1 to 7, 8 to 14, 15 to 21, 22 to 28, 29 to 35, 36 to 42, and 43 to 49.

Commencing with 1 from the north-west corner of the board, we follow the east-nor'-east knight-path, so that 2 falls at the base of the third column from the left on the board above. This in actual practice brings 2 in the given board at the base of the third column from the left. We then continue the knight-path, and on reaching 7, the end of the first sub-series, make a bishop's move, one square (cell) to the south-east for 8, and continue an east-nor'-east knight-path as before. On reaching 14, the end of the second sub-series, another south-east bishop's move of one square (cell) is made, and so we go on till all the forty-nine cells are filled. Of course, whenever, as in the case of 2 above, a knight's leap or bishop's move causes a term of the numerical series to fall outside the margin of the board, on to a cell of one of the similar contiguous boards, the term so falling must be transferred to its *corresponding cell within the margin*. Observe the terms 20, 21, and 4, for example, in Fig. G below. They would be transferred within the border to the cells indicated by the corresponding Roman numerals.

The Caïssan seven-square is perfect in all but certain of the knight-paths.

Fig. G shows the knight-paths which pass through the centre.

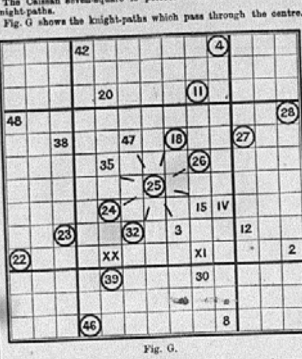


Fig. G.

They are all "magic" but the paths, which are, so to speak, parallel with the dark-ringed paths, that is to say the N.N.E. and E.N.E. paths are not magic. There are, then, in all, fourteen magic rook-paths, fourteen magic bishop-paths, and sixteen magic knight-paths in the square, each path summing 175. The central or mean term of the series, viz., 25, occupies the middle cell, and any two terms on opposite sides of it, equi-distant from it, and in the same straight line with it, are complementary, summing twice the mean—*i.e.*, 50. Note this in the knight-paths.

Now let us write the series 1—49 in the septenary scale, beginning with 0. The non-mathematical reader can easily do it, if it be remembered that no digit greater than 6 may be employed. The series then becomes 0, 1, &c., to 66. We give a portion of the series in line; but the whole of it will be found in the square, Fig. H.

0, 1 ......5, 6, 10, 11......15, 16, 20, 21......25, 26, 30, 31......66.

The arrangement shown in Fig. H may easily be obtained from

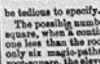
0_0	5_3	3_6	1_2	6_5	4_1	2_4
3_5	1_1	6_4	4_0	2_3	0_6	5_2
6_3	4_6	2_2	0_5	5_1	3_4	1_0
2_1	0_4	5_0	3_3	1_6	6_2	4_5
5_6	3_2	1_5	6_1	4_4	2_0	0_3
1_4	6_0	4_3	2_6	0_2	5_5	3_1
4_2	2_5	0_1	5_4	3_0	1_3	6_6

Fig. H.

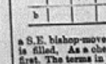
Fig. F by subtracting unity from every term, dividing it by 7, and placing quotient and remainder side by side, *e.g.* $(7-1) \div 7 = 06$. $(20-1) \div 7 = 25$.

The thick-faced left-hand digit is a multiple of the "radix" 7, just as in the common system of notation the left hand of the two digits represents so many tens. We shall call these thick digits radices.

It will now be seen that either the radices by themselves or the units by themselves form a magic square with the series 0 to 6 repeated seven times; also, that the unit-square is an exact counterpart of the radix-square turned as it were about its S.E. diagonal. If Caïssa be placed on any one of the forty-nine cells, at least ten magic-paths are open to her either in the radix-square or the unit-square. In other words, in at least ten ways she can pass through the digits 0, 1, 2, 3, 4, 5, 6. Hence there are at least seventy magic-paths summing 21, or, if 1 to 7 be taken, summing 28. Twenty-one of the paths are extra Caïssan paths of the form given in the diagram, K K. There are some others which it would be tedious to specify.



The possible number of magic paths from any cell of a prime-root square, when a continuous arithmetical series is employed, is always one less than the root; hence in the seven-square we get (as a rule) only six magic-paths from every starting point. The next prime-root-square, the eleven-square, having ten normal paths, can be made perfectly Caïssan. But the path of construction must not be a knight path. We shall call the construction-path, employed in Fig. J, an oblong bishop's path; it is shown by bb in the annexed diagram. It is compounded of a knight's leap, *plus* a one-square bishop's move. Starting from 1, we continue the oblong-bishop-path to the end of the first sub-series, *i.e.* 11, and then make a one-square bishop's move to the S.E., as in forming the seven-square. So, from 12 we continue the oblong-bishop-path to 22, the end of the second sub-series, make a S.E. bishop-move as before, and continue the process till the square is filled. As a check it will be well to complete the S.E. diagonal first. The terms in it will be 1, 13, 25, 37 and so on—a regular series increasing by 12. So from any term already placed we may descend diagonally to the south-east, increasing by 12, unless the term be the end of a sub-series, when we increase by 1 only, thus 6, 18, 30, 42, 54 [66], 67.



The next square, Fig. J, has at least one-hundred-and-ten magic paths, viz., twenty-two rook-paths, twenty-two bishop-paths, forty-four knight-paths, and twenty-two extra paths. It can be reduced to the radix 11, by either of the methods given under the seven-square, but we shall require a new digit for ten. Many mathematicians employ t; we find the Roman numeral X the most

1	83	44	115	76	[illegible]	108	58	40	99	51
63	13	95	45	6	88	38	120	70	31	102
114	75	25	107	57	18	89	50	11	82	43
55	5	87	37	119	69	30	101	62	12	94
106	56	17	99	49	10	81	42	113	74	24
36	118	68	29	100	61	22	93	54	4	86
98	48	9	80	41	112	73	23	105	66	16
28	110	60	21	92	53	3	85	35	117	67
79	40	111	72	33	104	65	15	97	47	8
20	91	52	2	84	34	116	77	27	109	59
71	32	103	64	14	96	46	7	78	39	121

Fig. J.

convenient. The series will now be 1 to 100, or 0o to Xx. (Observe, "100" is now eleven times eleven, or 121 in the ordinary notation.)

In Fig. K we give an eleven-square filled with a series having the radix eleven. Here, again, we have two interwoven Caïssan

0_0	7_5	3_X	X_4	6_9	2_3	9_8	5_2	1_7	8_1	4[illegible]
5_7	1_1	8_6	4_0	0_5	7_X	3_4	X_9	6_3	2_8	9_2
X_3	6_8	2_2	9_7	5_1	1_6	8_0	4_5	0_X	7_4	3_9
4_X	0_4	7_9	3_3	X_8	6_2	2_7	9_1	5_6	1_0	8_5
9_6	5_0	1_5	8_X	4_4	0_9	7_3	3_8	X_2	6_7	2_1
3_2	X_7	6_1	2_6	9_0	5_5	1_X	8_4	4_9	0_3	7_8
8_9	4_3	0_8	7_2	3_7	X_1	6_6	2_0	9_5	5_X	1_4
2_5	9_X	5_4	1_9	8_3	4_8	0_2	7_7	3_1	X_6	6_0
7_1	3_6	X_0	6_5	2_X	9_4	5_9	1_3	8_8	4_2	0_7
1_8	8_2	4_7	0_1	7_6	3_0	X_5	6_X	2_4	9_9	5_3
6_4	2_9	9_3	5_8	1_2	8_7	4_1	0_6	7_0	3_5	X_X

Fig. K.

repeated series squares, but now each is perfect. By employing this square first, the corresponding continuous-series square may be easily made.

In the west column the radices form a series increasing by 5 only; we must subtract 11 when getting beyond that number, *e.g.*, 0, 5, 10, 15 (but 15—11 = 4), 9, 14 (*i.e.*, 3), and so on. Having fixed these, the radices in a south-east direction run consecutively.

When the radix square is finished the unit square is easily made by repeating horizontally the arrangement which is perpendicular in the radix-square. Thus, as the west column of radices is **0, 5, X, 4, 9, 3, 8, 2, 7, 1, 6**, so the most northern unit row is 0, 5, X, 4, 9, 3, 8, 2, 7, 1, 6.

CAÏSSAN EVENLY-EVEN SQUARES.

In logical order, squares with odd composite roots should be mentioned next; but these, being the most difficult, are reserved to the last.

The eight-square given in *The Queen*, p. 142, shows the general method of constructing evenly-even squares; and if the series 1 to 64 or 0 to 63 be reduced to the radix 8, those who wish it can easily perceive the principle of construction.

For simplicity we take in Fig. L the series 0 to 7. Now $0 + 7 = 1 + 6 = 2 + 5 = 3 + 4$. Hence any two of these pairs

0_0	7_1	0_2	7_3	0_7	7_6	0_5	7_4
1_7	6_6	1_5	6_4	1_0	6_1	1_2	6_3
2_0	5_1	2_2	5_3	2_7	5_6	2_5	5_4
3_7	4_6	3_5	4_4	3_0	4_1	3_2	4_3
7_0	0_1	7_2	0_3	7_7	0_6	7_5	0_4
6_7	1_6	6_5	1_4	6_0	1_1	6_2	1_3
5_0	2_1	5_2	2_3	5_7	2_6	5_5	2_4
4_7	3_6	4_5	3_4	4_0	3_1	4_2	3_3

Fig. L.

are equal to any other two; also any pair multiplied by 4, equals the sum of the whole series.

In the eleven-square with *radix eleven*, it will be seen that in every magic path we have, both in the radices and in the units, every term of the series 0 to X. In the eight-square with radix *eight*, however, to take the radices, we have in the horizontal rows complementary pairs repeated, and in the perpendicular rows the whole series. Conversely with the units, we have in the horizontal rows every term of the series 0—7, and in the perpendicular rows repeated complementary pairs. Any four contiguous cells give, either in radices or units, and therefore in both combined, a constant sum. This suggests divisibility, and, with some modification and a slight sacrifice of path-giving capacity, these evenly-even root squares can be made frangible to a very high degree.

In Fig. M we have a Caïssan sixteen-square, which breaks up into four perfect Caïssan eight-squares. These eight-squares we shall, for distinction, call "squarelets." The two eastern may change places with the two western, the two northern may change

places with the two southern, the N.E. with the S.W., the N.W. with the S.E., without altering or impairing the properties of the

1	232	9	240	29	252	21	244	2	231	10	239	30	251	22	243
61	220	53	212	33	200	41	208	62	219	54	211	34	199	42	207
65	168	73	176	93	188	85	180	66	167	74	175	94	187	86	179
125	156	117	148	97	136	105	144	126	155	118	147	98	135	106	143
228	5	236	13	256	25	248	17	227	6	235	14	255	26	247	18
224	57	216	49	196	37	204	45	223	58	215	50	195	38	203	46
164	69	172	77	192	89	184	81	163	70	171	78	191	90	183	82
160	121	152	113	132	101	140	109	159	122	151	114	131	102	139	110
3	230	11	238	31	250	23	242	4	229	12	237	32	249	24	241
63	218	55	210	35	198	43	206	64	217	56	209	36	197	44	205
67	166	75	174	95	186	87	178	68	165	76	173	96	185	88	177
127	154	119	146	99	134	107	142	128	153	120	145	100	133	108	141
226	7	234	15	254	27	246	19	225	8	233	16	253	28	245	20
222	59	214	51	194	39	202	47	221	60	213	52	193	40	201	48
162	71	170	79	190	91	182	83	161	72	169	80	189	92	181	84
158	123	150	115	130	103	138	111	157	124	149	116	129	104	137	112

Fig. M.

whole square. Also, each of the squarelets may be turned through one, two, or three right angles (so long as its companions are treated in a precisely similar way) without injuring the great square. Again, if the squarelets be split into halves vertically, any right-half will, with any left-half, make a perfect Caïssan eight-square. The only essential point in which this differs from the ordinary Caïssan sixteen-square is that, in following a knight's path, eight cells must be taken in one squarelet before passing to another squarelet. The same rule applies in respect to the extra paths.

The method of constructing this square is extremely simple. Having ruled the lines for a sixteen-square, divide it into four squarelets, placing 1, 2, 3, 4 in natural-square order in the corners of the squarelets. These become the index numbers. Fill No. 1 squarelet with the light series, 1, 5, 9, &c., increasing by four, and with the heavy complementary series, 256, 252, 248, &c., decreasing by four in the order given in Figs. L, M, N, &c. When No. 1 squarelet is completed, make every term of the light series in No. 2 squarelet *greater by* 1, and every term of the heavy series *less by* 1, than the corresponding term in No. 1 squarelet. Each term in squarelet No. 3 will, of course, be *greater* or *less* by 2, each term in No. 4 squarelet *greater or less* by 3, though it is easier to construct No. 4 from No. 3.

By sacrificing the knight-paths we can get a still greater amount of frangibility. In Fig. N we have the "Caïssan Square of Squares," breaking up into sixteen squarelets. It has the following

1	224	49	240	2	223	50	239	3	222	51	238	4	221	52	237
113	176	65	160	114	175	66	159	115	174	67	158	116	173	68	157
208	17	256	33	207	18	255	34	206	19	254	35	205	20	253	36
192	97	144	81	191	98	143	82	190	99	142	83	189	100	141	84
5	220	53	236	6	219	54	235	7	218	55	234	8	217	56	233
117	172	69	156	118	171	70	155	119	170	71	154	120	169	72	153
204	21	252	37	203	22	251	38	202	23	250	39	201	24	249	40
188	101	140	85	187	102	139	86	186	103	138	87	185	104	137	88
9	216	57	232	10	215	58	231	11	214	59	230	12	213	60	229
121	168	73	152	122	167	74	151	123	166	75	150	124	165	76	149
200	25	248	41	199	26	247	42	198	27	246	43	197	28	245	44
184	105	136	89	183	106	135	90	182	107	134	91	181	108	133	92
13	212	61	228	14	211	62	227	15	210	63	226	16	209	64	225
125	164	77	148	126	163	78	147	127	162	79	146	128	161	80	145
196	29	244	45	195	30	243	46	194	31	242	47	193	32	241	48
180	109	132	93	179	110	131	94	178	111	130	95	177	112	129	96

Fig. N.

properties: All the properties, except the knight-paths of the Caïssan sixteen-square, and all the properties, with that exception of the square given in Fig. M. Besides these, as the squarelets stand, any contiguous four of them will make a Caïssan eight-square, and any contiguous nine will form a Caïssan twelve-square. In fact, twelve-squares and eight-squares may be made in endless variety, by merely noticing the index numbers of the squarelets.

Thus, if the indices are—

2	4
6	8

or

3	6
9	12

we shall get Caïssan eight squares, for $2 + 8 = 6 + 4$, and $3 + 12 = 9 + 6$. So, for example, if the indices are—

1	3	5
6	8	10
11	13	15

we shall have a twelve-square, because $11 + 8 + 5 = 13 + 10 + 1 = 15 + 6 + 3 = 24$.

The "Caïssan Square of Squares" has, further, the properties of a ringed-square. By "ring" we mean a group of cells forming a hollow square, as in the annexed diagram. Of course we can strip off the twelve external squarelets, leaving an eight-square; but, besides this, we can strip off the outer ring of cells, and the next but one; then, taking out the numbers, 17, 65, 173, 253, 164, 244, 32, and 80 in the dark cells, *fit the remainder of that ring* to the inner square so as to have a Caïssan twelve-square. The process may be repeated with this twelve-square. This remarkable square of squares has many other singular properties, as any one may ascertain by trying experiments with it. It is constructed on the same principle as that given in Fig. M. Squarelet No. 1 is filled with the light series 1, 17, 33, &c., increasing by 16, and with the heavy complementary series 256, 240, 224, &c., decreasing by 16.

The terms in No. 2 are made each *greater or less* by 1, those in No. 3 greater or less by 2, and so on till all are filled.

(To be continued.)

The Queen, the Lady's Newspaper & Court Chronicle
vol. 70, p. 391 (October 15, 1881), reel 20

``Caïssan magic squares'' by ``Ursus'' (part 3/3)

PASTIMES.

TO CORRESPONDENTS.

*** Forms for Solvers (to prevent the rejection of solutions through informality) may be had of The Publisher, *The Queen*, 346, Strand, London, W.C., price 1d. per dozen; by post 1½d. per dozen.

*** Rules for "Queen" Double Acrostic Prizes will be forwarded on receipt of a stamped envelope, addressed to The Publisher, *The Queen*, 346, Strand, London, W.C.

Itti Duffa; J. E. W.—We have carefully considered your alternatives, but cannot see that they satisfy the lights.

Pizzie had "rorarii" instead of "rorarii" in No. 722.

Worry Worry.—A reasonable number of alternatives are allowed.

Black Bess (Miss H.)—Please choose another pseudonym, "Black Bess" being already appropriated.

*** Numerous solutions are excluded from omission of name and address.

A Villager.—We have published from time to time similar queries, especially about the end of the year, without obtaining any replies of moment. Send a stamped directed envelope for list of plays suitable for private performance.

DOUBLE ACROSTIC No. 728.

To former, my latter is sent for Alphonso,
And Nordenheim journeys to fasten it on, so
Each insular prejudice let us now banish,
Let true British subjects shake hands with the Spanish.

1. A big 'un in the City,
You may find me in the fog.
And—O solvers wise and witty—
I am more than quite agog.

2. A weed am I but not to smoke,
I make a 'bacca nalian joke:
This definition's far too lax,
For I have ties with hemp and flax.

3. No tiring maid am I, to grace
With stylish robes my lady fair;
For in the kitchen is my place,
'Mong pots and pans and crockery-ware

4. It was at Margate last July,
I met this vulgar boy:
He shed a tear, he heaved a sigh,
A stranger seemed to joy;
I changed his name, in brief express'd,
(He was a vulgar child).
I said "a pint of very best,
And please to draw it mild."

5 and 6. Isis, ere she had that name,
Ate my sixth; now Argus went,
Saw the lady do the same,
Saw my fifth, this element.

Ursus.

SOLUTION OF DOUBLE ACROSTIC No. 726.

[First of the Quarter.]

P	PATTI	I
O	OPTION	N
L	LASHES	S
Y	YULE-FEAST *	T
T	TURNERELLI	I
E	EIGHT †	T
C	CADEAU	U
H	HE-RAT	T
N	NEROLI	I
I	IN-I-GO	O
C	CALD(E)RON	N

Ursus.

* You'll feast. † Eight, ait or eyot, all pronounced in the same way, a small island. Eight, a crew of eight with coxswain.

[Correct replies have been received from All of Us, Addie, Alphabeta, Argus, Athenian, Athelstone, Armorie, Alma, Absentee, Alf, A Tory, Adrem, A. B., Aibey, A Pilgrim, A J. S., Aunt Tab, Amethyst, A. B. C., Amenophis, Boggart, Blue, Beauly, Barryfield, Ba-Ba, Beatrice W., Badshahzadi, Beppo, Burwains, Bayton, Black Bess, Bobbins and Co., Black Bess (Miss H.), Chloe, Calais, Clara, Charon, Crux, Court Lodge, Chattisgurh, Cerocceil, Chameleon, Clef, Crème de Joné, Chamois, Charta, Donus, Dust, Domum, Dedham, Dandylit, Darby and Joan, East-Anglian, Elais, Eily, Ebboo, Enfield, E. C., E. V. P., Folly, Faw, Fairley, Fizzie, Francie, Fiesole, Firenza, Frankton, Fitz, Gallicap, Gertrude, Gwyr, Good Luck's Sister, Gunner's Wife, Guildroy and Co., Gippy, Hazel Mount, I. V. F., Itti Duffa, Isabel, Jim and Sid, J. E. W., Jumbo, Kelt, Kangaroo, Katydid, Kingswood, Kilah Bilah, Kate Leicester, Lex, Leander, L. L., Lynx, Legion, Little Nell, Mater, Milly Muff, Mambo and Pollie, May, Mistress Maria and E.M.B.S., Mitz, Mignonette, Minnehaha, Mem, Manirette, Mus-Bac-Oxon, Mrs Mums, Milton, Mandiana, Nil Desperandum, Naiad, Non Timeo, O.W., Ovingdean, Oakwood, Owl, Petunia, Patmore, Phantasmagoria, Pollin, Pitt, Psycho, P.H.E., Paddie, Pangbourne Naish, Peakie, Rosa Bonheur, Rags and Tatters, Ronille, Rueville, Rochsoles, Rain Bird, Reldas, Respice Finem, Shenstone, Smashjavelin, Sidmouth, Son A., Semper Idem, Speldhurst, Three Witches, The Child, Tricycle, The Basonikon, Trot, Tommy Tumps, Trienant, The Grosvenors, Tut-o-mum, Undine, Vier Mädchen, Vancluse, Winkle, West Chinnock, W. M., Work, Worry Worry, Woodstock, Whiskey, Whinmoor, Zebra, Zep. All others are wrong or do not conform to rules.—Ed.]

DOUBLE ACROSTIC PRIZE.

Full Score for the Quarter ended last Saturday in September, 1881.

Correct solutions.

Emilie [Mrs Webster, 41, Bayston-road, Stoke Newington.] . . 12
Pollin (cannot take another quarterly prize)
A. J. S., Phantasmagoria, Respice Finem, Zebra 11
Blue, Chattisgurh, Charon, Calais, E. C., J. E. W., Kelt, Katydid, O. W. 10
Brentry, Ebboo, Mrs Mums, Sea Serpent, Tut-o-mum 9
Ba-Ba, Beppo, Barryfield, Badshahzadi, Dedham, Francie, Fizzie, Kate Leicester, L. L., Mitz, Oakwood, P. H. E., Smashjavelin, Shenstone 8
Beatrice W., Enfield, E. V. P., May, Mistress Maria and E. M. B. S., Psycho 7
Burwains, Cypher, E. J. C., Little Nell. 6
Bayton, Iris, Itti Duffa, Jumbo, Mignonette, Rehoboth, Speldhurst, Trenant, Try-try-again, Winkle 5
A. B., Croydon Cat, Gad-about, Hampton Courtier, Jeux St. George, Pangbourne Naish, Printemps, Rueville, Two Guernseyites, Tommy Tumps, W. M. 4
Alma, Corrie, Dust, Kingswood, Louise, Naiad, Non Timeo, Ovingdean, Owl, Patmore, Peakie, Suffolk Dumplings, Sidmouth, Vancluse, Wellwood, Zep. 3
Amethyst, Bogantourie, Cat, Dog and Co., Dumfoodle, Golden Plover, Gippy, Rags and Tatters, Undine, Veilchen, Vier Mädchen 2
Ashbury, A. R. A., Anim, Argus, Bruce, Charlton, Certosa, Chalgrove, Daisy, Dunheved, Elais, Flora Macdonald, Georgie and Francie, Haram Trebreh, Kiddy, Kilah Bilah, Lex, Muff, Mexican, Mandiana, Minnehaha, Mary Anne, Petunia, Tweedledum and Tweedledee, Tottie, The Grosvenors, Three Witches, Trident, Tiny, Van, Whiskey 1

Corrections, if required, will be received till first postal delivery, Tuesday, Oct. 25.

NOTES AND QUERIES.

TABLEAUX VIVANTS.—Suggestions wanted for dresses representing the seasons, for tableaux vivants.—Autumn.

CHARACTERS FOR FANCY FAIR.—Could you kindly oblige me with a list of characters suitable for a young lady to appear in at a fancy fair? I should also feel obliged if you could recommend me any book on the subject.—Eunice.

ANSWERS.

THINGS FOR BURYING.—Cigar, up comes a weed. Leg of mutton, up comes (h) ash. Boat, up comes (h) elm. Refused suitor, up comes love-lies-bleeding. Sea shore, up comes beach (beech). Schoolmaster's rod, up comes a birch. A lady's satin shoes, up come lady's slippers. Mr Gladstone, up comes sweet william. Dog, up comes a bough. A wise man, up comes sage.—Anon.

—— A few suggestions to "Surrey" for burying, or planting things, are these: Frog—hops; cathedral tower—Canterbury bells; bark—hound's-tongue; tobacco-pipe—fumitory; Scotch river—yarrow; rainbow—iris; shell-fish—periwinkle; fortune-hunter—mary-gold; Chinese pig-o-tee.—Harpra.

CAÏSSAN MAGIC SQUARES.

By "URSUS."

(*Concluded from The Queen of Sept. 10.*)

Caïssan Squares with Oddly-even Roots.

These cannot be made absolutely perfect. We may make them with mutilated series, as 1, 2, 3, 4, 5, 7, &c., or imperfectly Caïssan, as in Fig. P.

99	08	+02	+93	+04	*10	92	03	94	90
80	11	82	13	84	18	87	16	85	19
79	28	77	26	75	21	72	23	74	20
60	31	62	33	64	38	67	36	65	39
59	48	57	46	55	41	52	43	54	40
*1	81	12	83	14	88	17	86	15	98
29	78	27	76	25	71	22	73	24	70
30	61	32	63	34	68	37	66	35	69
49	58	47	56	45	51	42	53	44	50
09	91	97	06	95	89	07	96	05	00

Fig. P.

Here we have a perfect magic square having rook-paths and diagonals with constant summations, and on removing the outer ring it becomes a perfect Caïssan square.

To construct it, take the series 1 to 100, or, to show more clearly the principle of formation, the series 0 to 99. Throw out for the ring 0 to 9, 90 to 99 and all the terms having either 9 or 0 in the unit's place. In a "natural square" these fall in the outer ring. Fill the centre eight-square with the remaining terms 11 to 48 for the light series, 88 to 51 for the heavy series, as in the preceding squares. At the corners of the ring place 0, 9, 90, and 99 in inverted natural square order. In the north and in the west cell-rows place the terms which would be there if the law evident in the central board prevailed throughout. Place opposite each of these terms, on the further side of the ring (*i.e.*, in the south and east cell rows), its complementary term. Then transpose 1 and 10, the terms marked by the asterisk, and instead of the terms 97, 6, and 95 place 2, 93, 4, their complementary terms as shown by the crosses. The square will then be complete as in Fig. P.

Caïssan Squares with Odd Composite Roots.

These require special constructions, but yield a large number of magic paths. Thus the fifteen-square has at least three hundred and twenty-four distinct magic paths. It is given in Fig. Q:

129	6	207	131	2	204	126	12	206	122	9	201	132	11	197
76	193	68	78	195	61	88	188	63	90	181	73	83	183	75
179	110	49	175	112	59	170	109	55	172	119	50	169	115	52
159	36	147	161	32	144	156	42	146	152	39	141	162	41	137
16	223	98	18	225	91	28	218	93	30	211	103	23	213	105
134	5	199	130	7	209	125	4	205	127	14	200	124	10	202
84	186	72	86	182	69	81	192	71	77	189	66	87	191	62
166	118	53	168	120	46	178	113	48	180	106	58	173	108	60
164	35	139	160	37	149	155	34	145	157	44	140	154	40	142
24	216	102	26	212	99	21	222	101	17	219	96	27	221	92
121	13	203	123	15	196	133	8	198	135	1	208	128	3	210
89	185	64	85	187	74	80	184	70	82	194	65	79	190	67
174	111	57	176	107	54	171	117	56	167	114	51	177	116	47
151	43	143	153	45	136	163	38	138	165	31	148	158	33	150
29	215	94	25	217	104	20	214	100	22	224	95	19	220	97

Fig. Q.

And in Fig. R we show the extra Caïssan paths, the ordinary ones being taken for granted. If Caïssa be placed on the cell marked A, she can move in the direction A B, A b, A C, A c, and so on to

			*		f									
	*				e		+							
					d									*
		+			c	+						*		
					b					*				
*			+		+			*						
		*				*	g		h	k				
				A										
		*				*	G		H	K				
*			+		+			*						
					B				x_1	*				
		+			C	+						*		
					D		y_1							*
	*				E		+							
			*		F									

Fig. R.

make magic paths. Also, if A happens to be the central cell, the construction paths Ax, Ay, and the knight's path shown by the asterisks, will also be magic.

The construction is not so difficult as might be imagined:

A	b	c	d	e
d	e	a	b	c
b	c	Dd	e	a
e	a	b	Cc	d
c	d	e	a	Bb

It will have been noticed that in a prime root square it is very easy to make each path contain every term of a repeated series; and if we have two series interwoven, then each path must contain every term of both one series and the other. Hence, if we have a series representing the multiples of the radix, and another series representing the units less than the radix, each path is bound to give us the sum required. The most apparent cases are those of the S.E. diagonals of the seven-square and the eleven-square, where we have O_0, 1_1, 2_2, 3_3, &c., just as in the accompanying diagram we have Aa, Ee, Dd, &c. The order of the terms is quite immaterial, so long as the same order is maintained throughout. In composite root-squares, however, we have certain complementary cell-groups repeated, as 0, 7; 1, 6; 2, 5; 3, 4; in the eight-square. Fig. S., illustrating the principle of the fifteen-square, will furnish another example.

Hh	Oe	Mk	Hj	Oa	Mh	He	Ok	Mj	Ha	Oh	Me	Hk	Oj	Ma
Eo	Ll	Dg	Eb	Ln	Do	El	Lg	Db	En	Lo	Dl	Eg	Lb	Dn
Km	Gd	Cc	Ki	Gf	Cm	Kd	Gc	Ci	Kf	Gm	Cd	Kc	Gi	Cf
Jh	Be	Ik	Jj	Ba	Ih	Je	Bk	Ij	Ja	Bh	Ie	Jk	Bj	Ia
Ao	Nl	Fg	Ab	Nn	Fo	Al	Ng	Fb	Aa	No	Fl	Ag	Nb	Fn
Hm	Od	Mc	Hi	Of	Mm	Hd	Oc	Mi	Hf	Om	Md	Hc	Oi	Mf
Eh	Le	Dk	Ej	La	Dh	Ee	Lk	Dj	Ea	Lh	De	Ek	Lj	Da
Ko	Gl	Cg	Kb	Gn	Co	Kl	Gg	Cb	Kn	Go	Cl	Kg	Gb	Cn
Jm	Bd	Ic	Ji	Bf	Im	Jd	Bc	Ii	Jf	Bm	Id	Jc	Bi	If
Ah	Ne	Fk	Aj	Na	Fh	Ae	Nk	Fj	Aa	Nh	Fe	Ak	Nj	Fa
Ho	Ol	Mg	Hb	On	Mo	Hl	Og	Mb	Hn	Oo	Ml	Hg	Ob	Mn
Em	Ld	Dc	Ei	Lf	Dm	Ed	Lc	Di	Ef	Lm	Dd	Ec	Li	Df
Kh	Ge	Ck	Kj	Ga	Ch	Ke	Gk	Cj	Ka	Gh	Ce	Kk	Gj	Ca
Jo	Bl	Ig	Jb	Bn	Io	Jl	Bg	Ib	Jn	Bo	Il	Jg	Bb	In
Am	Nd	Fc	Ai	Nf	Fm	Ad	Nc	Fi	Af	Nm	Fd	Ac	Ni	Ff

Fig. S.

Having no digit greater than 9, let the letters A to O consecutively stand for the series O to 14, in the following order; for the units the small letters:

a, b, c, d, e, f, g, h, i, j, k, l, m, n, o
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 0

and for the radices (strictly multiples of the radix, 15), the capitals:

A, B, C, D, E, F, G, H, I, J, K, L, M, N, O
15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195, 210, 0

Now the sum of the series o ..... n, is 15 (o + n) ÷ 2 = 105, but in virtue of being a multiple of 3 (greater than twice 3), it breaks up into triplets, each triplet having a constant sum; also, in virtue of being a multiple of 5 (greater than twice 5), it breaks up into five-term groups also having a constant sum. Hence (omitting the *plus* signs for the sake of compactness):

hom = eld = kgc = jbi = anf = 21
and hekja = olgbn = mdcif = 35

So that, to take examples:

5 hom = 3 hekja = o ..... n.

That is, five repeated triplets, or three repeated five-groups, give the same sum as the whole series.

So, h e k j a, *i.e.*, 8, 5, 11, 10, 1 = 35
o l g b n — 0, 12, 7, 2, 14 = 35
m d c i f — 13, 4, 3, 9, 6 = 35
21, 21, 21, 21, 21 = 105

forms a parallelogram, each of the long rows summing one-third of the whole series, and each of the short rows one-fifth of the series.

For Girtonians and Newnhamites we may state the general rule thus:—If the series consists of mn terms, it will break up into n groups (of m in a group), each summing $\frac{1}{n}$ of the whole series, and also into m groups (of n in a group), each summing $\frac{1}{m}$ of the whole series.

Of course mn must be either odd or evenly-even.

This reasoning applies equally to the radix-series, for the sum of that is merely fifteen times the sum of the units series. Now, let S, s stand for the sum of the whole series, T, t for the sum of a triplet, and V, v for the sum of a five-group, the capitals, of course, representing radices. Then, starting from the S.W. corner, we have the N. rook path summing:

3 V + 5 t = 1575 + 105 = 1680

the E. rook path:

5 T + 3 v = 1575 + 105 = 1680

a N.E. bishop path:

S + s = 1575 + 105 = 1680

an E.N.E. knight-path, indicated by the dark cells:

S + s = 1575 + 105 = 1680

and so on with the other paths. In the numbered square, Fig. Q, we have added unity to each cell, so as to make the series 1-15, and the sum S + s = 1695.

For a twenty-one square we should require triplets and seven-groups; the object of this double system of grouping is the prevention of a recurrence of any particular combination, *e.g.*, in our figure the combination A m occurs in the S.W. corner only.

It may now be seen why oddly-even roots fail to give Caïssan squares. Take as an example the root 6. The series 1—6 should, the sum being a multiple of 3, break into triplets; but this it will not do. In common magic squares the excess of one triplet may, by trial, be made to balance the defect of another, but even in these an oddly-even square in which there is a regular law is a mathematical impossibility.

The idea of the foregoing Caïssan magic squares was suggested partly by the Brahminical squares, that from time immemorial have been used by the Hindoos as talismans; partly by an article by the Rev. A. H. Frost, M.A., of St. John's College, Camb., on "Nasik Squares," in No. 57 of the "Quarterly Journal of Pure and Applied Mathematics" (1877). Our squares are, however, mostly original, as are the methods of construction, though one or two, notably that for the fifteen-square, may by Girtonians and Newnhamites readily be translated into the (mathematically) purer language adopted in Mr Frost's able paper. The epithet "Caïssan" describes the distinguishing characteristics of the squares, as the paths include all possible continuous chess moves. We commenced with Caïssa's mythical history; let us conclude with a few facts less apocryphal. We have alluded to the Brahminical squares. Of these the favorites are the four-square and the eight-square. We have shown that the first perfect one; in fact, as the Policeman of Penzance would say, "taking one consideration with another," it may be pronounced the best of the lot. Now, *chaturanga*, the great-grandfather of chess, is of unknown antiquity, and Duncan Forbes gives translations of venerable Sanskrit manuscripts, which describe the pieces, moves, and mode of playing the game. *Chatur anga*, means four arms, *i.e.*, the four parts of an army—elephants, camels, boats (for fighting on the rivers, Sanskrit *roka*, a boat; hence rook), and foot-soldiers, the whole commanded by the kaisar or king. At first there were four armies, two as allies against two others; afterwards the allies united, one of the kings becoming prime minister with almost unlimited powers, and in the chivalrous west being denominated Queen. We have already noticed that the Russians, who are inveterate chess players, and who probably got their national game direct from the fountain head, endow their queen with the powers of all the other pieces. So then in the west we have king, queen, bishop (ex-elephant), knight (ex-camel), castle or rook, and pawn, the line soldier on whom, after all, military success depends. Sir William Jones, who wrote "Caïssa" when he was a boy, thinking in Greek, but then little versed in Sanskrit, turned the corrupt Saxon "chess" into the pseudo-classic Caïssa. Strange coincidence, for much later, his admirer, Duncan Forbes, traces "chess," by such links as the French *échecs*, English *check* and *check mate*, German *schach matt*, to the Persian *Shah mat*: Shah being their rendering of the Sanskrit or Aryan *Kaisar*, still retained by Germans, and by Russians in *Czar*.

Sir William's Caïssa would pass very well for the feminine of Kaisar; but this is not the coincidence we wish to accentuate.

Both "chess" and "Caïssan" squares, under whatever name the reader pleases, have been known in India—the nursery of civilisation—from the remotest antiquity; hence there would seem to be a close connection between them. India is now the greatest possession of the British crown, and our Queen, as everyone knows, bears the masculine title of Kaisar-in-Hind.

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