

Problem Solving seminar. Team selection contest 2017.

1. An investor buys stock worth \$10,000 and holds it for n business days. Each day he has an equal chance of either gaining 2% or losing 1%. However in the case he gains every day (i.e. n gains of 2%), he is deemed to have lost all his money, because he must have been involved with insider trading. Find a simple formula of the amount of money he will have on average at the end of the n days.

2. Find the minimum value of

$$\frac{(x + 1/x)^6 - (x^6 + 1/x^6) - 2}{(x + 1/x)^3 + (x^3 + 1/x^3)}$$

for $x > 0$.

3. Prove that a triangle in the plane whose vertices have integer coordinates cannot be equilateral.

4. Let $p(x)$ be a polynomial that is nonnegative for all real x . Prove that for some k , there are polynomials $f_1(x), \dots, f_k(x)$ such that

$$p(x) = \sum_{j=1}^k (f_j(x))^2.$$

5. For a positive integer a , let $P(a)$ denote the largest prime divisor of $a^2 + 1$. Prove that there exist infinitely many triples (a, b, c) of distinct positive integers such that $P(a) = P(b) = P(c)$.

6. What is the rightmost digit of

$$\left\lfloor \frac{10^{20000}}{10^{100} + 3} \right\rfloor ?$$