

Name \_\_\_\_\_

**Problem Solving seminar - Team selection contest**  
Friday September 28th, 2012, 3 hours

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Write down only your final solutions in this booklet. Justify your answers.

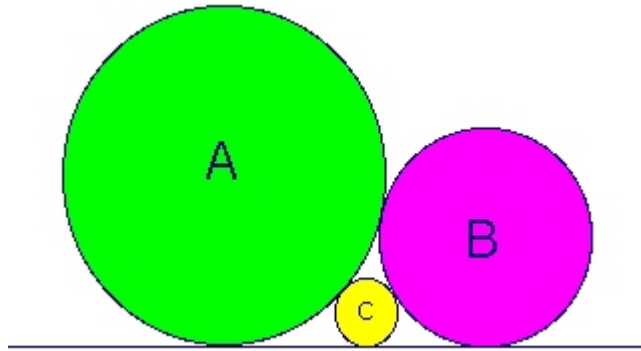
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Problem	Your score
1	
2	
3	
4	
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6	
Total	

1. Three pasture fields have areas of  $10/3$ , 10 and 24 acres, respectively. The fields initially are covered with grass of the same thickness and new grass grows on each at the same rate per acre. If 12 cows eat the first field bare in 4 weeks and 21 cows eat the second field bare in 9 weeks, how many cows will eat the third field bare in 18 weeks? Assume that all cows eat at the same rate.

2. Find

$$\sum_{n=1}^{\infty} \frac{1}{n4^n}.$$



- 3.** Two circles  $A$  and  $B$  with radii 2 and 1 are placed so that they are tangent to each other and a straight line. A third circle  $C$  is nestled between them so that it is tangent to the first two circles and the line. Find the radius of the third circle.

4. The function  $f(n)$  is defined for all positive integers  $n$  and takes on non-negative integer values. Also, for all  $m, n$

$$f(m+n) - f(m) - f(n) = 0 \text{ or } 1,$$

$$f(2) = 0, f(3) > 0, \text{ and } f(9999) = 3333.$$

Determine  $f(2012)$ .

5. Given a set  $S$  of 2012 distinct positive integers, none of which has a prime divisor greater than 20. Prove that  $S$  contains at least one subset of four distinct elements whose product is the fourth power of an integer.

6. Find the third digit after the decimal point of

$$(2 + \sqrt{5})^{100} \left( (1 + \sqrt{2})^{100} + (1 + \sqrt{2})^{-100} \right).$$

[For example, the third digit after the decimal point of  $\pi = 3.14159\dots$  is 1.]