MATH 350: Graph Theory and Combinatorics. Fall 2015.
Due in class on Friday, December 4th.

Assignment #5: Series-parallel and Planar graphs.

1. A graph G is *outerplanar* if it can be drawn in the plane so that every vertex is incident with the infinite region. Show that a graph G is outerplanar if and only if G has no K_4 or $K_{2,3}$ minor. (*Hint:* Add a vertex appropriately and use Kuratowski's theorem.)

2.

a) Show that every series-parallel graph is planar.

b) Is every series-parallel graph outerplanar?

c) What is the maximum possible number of edges in a simple series-parallel graph with n vertices?

3. Let G be a loopless graph, such that G does not contain $K_{2,3}$ as a minor. Show that either $\chi(G) \leq 3$, or G contains K_4 as a subgraph.

4. Let G be a graph drawn in the plane. Suppose that there exists a vertex v so that v belongs to the boundary of every region. Show that

$$\alpha(G) \geq \frac{1}{2}(|V(G)| - 1).$$

5. Let G be drawn in the plane so that

- the boundary of the infinite region is some cycle C,
- every other region has boundary a cycle of length 3, and
- every vertex of G not in C has even degree.

Show that $\chi(G) \leq 3$. (*Hint:* Use induction. For the induction step consider two cases: whether some two non-consecutive vertices of C are adjacent. In the second case, delete an edge of C and apply the induction hypothesis.)