MATH 350: Graph Theory and Combinatorics. Fall 2015.
Due in class on Friday, November 20th.

Assignment #4: Ramsey theorem, matching and vertex coloring.

1. Show that $R(3,4) = 9$.

2. Let

$$r_k := R_k(\underbrace{3, 3, \ldots, 3}_{k \text{ times}}).$$

(I.e. r_k is the minimum integer $n > 0$ such that every coloring of edges of K_n in k colors is guaranteed to produce a monochromatic triangle.) Show that

$$r_k \leq k(r_{k-1} - 1) + 2$$

for $k \geq 2$.

3. Let G be a graph and $Z \subseteq V(G)$. Show that the following are equivalent:

(i) G has a matching covering Z, and

(ii) for every $X \subseteq V(G)$ there are at most $|X|$ odd components C of $G \backslash X$ such that $V(C) \subseteq Z$.

4. Show that if G is a loopless graph, $k \geq 1$ is an integer and $\chi(G) > k$ then G has a path with k edges.

5. Let G be a loopless graph with $\chi(G) = k$ for some positive integer k. Show that G contains at least k vertices with degree at least $k - 1$.