MATH 350: Graph Theory and Combinatorics. Fall 2015.
Due in class on Friday, November 6th.

Assignment #3: Menger's theorem, vertex covers and network flows.

1. Show that $\tau(G) \leq \frac{1}{2}(|E(G)| + 1)$ for every connected graph G.

2. Let v be a vertex in a 2-connected graph G. Show that v has a neighbor u such that $G \setminus u \setminus v$ is connected.

3. Let G be a connected graph in which every vertex has degree three. Show that if G has no cut-edge then every two edges of G lie on a common cycle.

4.

a) Distinct $u, v \in V(G)$ are *k-linked* if there are k paths $P_1, ..., P_k$ of G from u to v so that $E(P_i \cap P_j) = \emptyset$ $(1 \leq i < j \leq k)$. Suppose u, v, w are distinct and u, v are k-linked, and so are v, w. Does it follow that u, w are k-linked?

b) Subsets $X, Y \subseteq V(G)$ are *k-joined* if $|X| = |Y| = k$ and there are k paths $P_1, ..., P_k$ of G from X to Y so that $V(P_i \cap P_j) = \emptyset$ $(1 \leq i < j \leq k)$. Suppose $X, Y, Z \subseteq V(G)$ and X, Y are k-joined, and so are Y, Z. Does it follow that X, Z are k-joined?

5. Let G be a directed graph and for each edge e let $\phi(e) \geq 0$ be an integer, so that for every vertex v,

$$\sum_{e \in \delta^-(v)} \phi(e) = \sum_{e \in \delta^+(v)} \phi(e)$$

Show there is a list $C_1, ..., C_n$ of directed cycles (possibly with repetition) so that for every edge e of G,

$$|\{i : 1 \leq i \leq n,\ e \in E(C_i)\}| = \phi(e).$$