MATH 350: Graph Theory and Combinatorics. Fall 2015.
Due in class on Friday, October 16th.

Assignment #2: Spanning trees, bipartite graphs and matchings.

1. We say that $F \subseteq E(G)$ is *even-degree* if every vertex of G is incident with an even number of non-loop edges in X. Show that if T is a spanning tree of G, there is an even-degree set $F \subseteq E(G)$ with $F \cup E(T) = E(G)$. (*Hint*: First, show that if F_1 and F_2 are both even-degree then so is $F_1 \triangle F_2 := (F_1 - F_2) \cup (F_2 - F_1)$.)

2. Show that a graph G is bipartite if and only if $\alpha(H) \geq |V(H)|/2$ for every subgraph H of G.

3. Let $k \geq 3$ be an integer. Let G be a bipartite graph such that

$$3 \leq \deg(v) \leq k \qquad \text{for every } v \in V(G).$$

Show that G contains a matching of size at least $\frac{3|V(G)|}{2k}$.

4. Let G be a bipartite graph with bipartition (A, B) in which every vertex has degree ≥ 1. Assume that for every edge of G with ends $a \in A$ and $b \in B$ we have $\deg(a) \geq \deg(b)$. Show that there exists a matching in G covering A.

5. Given integers $n \geq m \geq k \geq 0$, determine the maximum possible number of edges in a simple bipartite graph G with bipartition (A, B), with $|A| = n, |B| = m$ and no matching of size k.