Name _____

MATH 350: Graph Theory and Combinatorics. Fall 2012 - Midterm Exam

Thursday, October 11th, 2012, 16:35-17:25

The questions have to be answered in the booklets provided. Write your answers clearly. Justify all your answers. You can consult your notes and textbooks. Use of calculators, computers, cell-phones, etc. is not permitted.

Problem	Your score
1	
2	
3	
Total	

1. Let G be a simple graph with $|V(G)| = 8$ and $\deg(v) = 4$ for every $v \in V(G)$.

a) Does G necessarily contain a closed walk using every edge exactly once (i.e. *an Eulerian tour*)?

b) Does G necessarily contain a Hamiltonian cycle?

c) Is G necessarily 3-connected?

2. Let G be a bipartite graph with bipartition (A, B). Suppose that for every $X \subseteq A$ there exist at least $|X| - 1$ vertices of B with a neighbor in X. Show that G contains a matching of size $|A| - 1$.

3. Let G be a graph, let s, t be distinct non-adjacent vertices of G, and let $k \geq 1$ be an integer. Suppose that there exist paths $Q_1, Q_2, \ldots, Q_{2k-1}$ in G, each with ends s and t, such that no vertex in $V(G) - \{s, t\}$ belongs to more than two of these paths. Show that there exist paths $P_1, P_2, \ldots, P_k$ in G, pairwise disjoint, except for their ends s and t.