Q1 Let G be the graph pictured on Figure 1.

    a) Is G planar?

    b) What is the maximum integer k, such that G is k-connected?

    c) Find $\chi(G)$.

    d) Find $\chi'(G)$.

Q2 Let G be the graph with weights $w : E(G) \to \mathbb{Z}_+$ be the graph pictured on Figure 2. (*Ignore the edge directions for this problem.*)

    a) Find the min-cost spanning tree in G.

    b) Find a shortest path spanning tree for the vertex S.

Q3 Let G be the digraph pictured on Figure 2 with capacity $c(e)$ indicated for every edge. Find the c-admissible (S,T)-flow in G of maximum total value.

Q4 Let G be a simple graph with $2m$ vertices that has exactly one perfect matching. Prove that $|E(G)| \leq m^2$.

Q5 **a)** Show that every series-parallel graph is planar.

    b) Is it true that every series-parallel graph can be drawn in the plane so that all the vertices belong to the boundary of a single region?

    c) What is the maximum possible number of edges in a simple series-parallel graph with n vertices?

Q6 Show that a graph G is perfect if and only if every non-empty induced subgraph H of G contains a clique $S \subseteq V(H)$ such that $\alpha(H \setminus S) < \alpha(H)$.
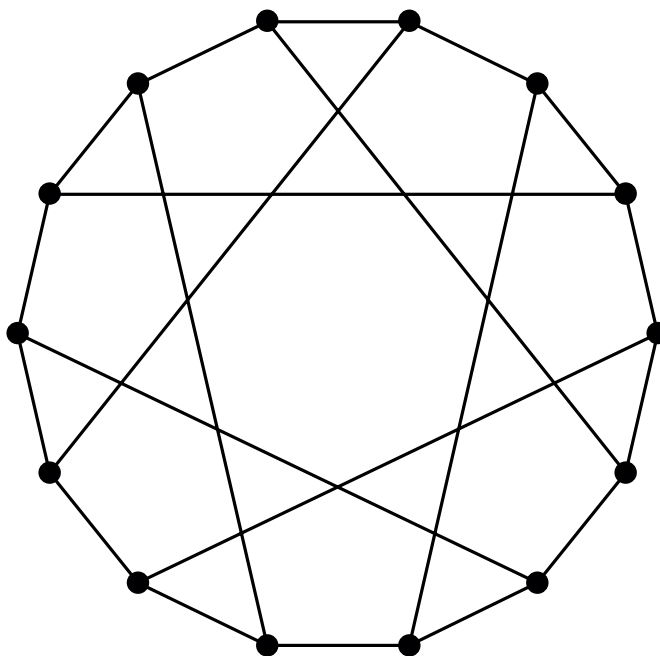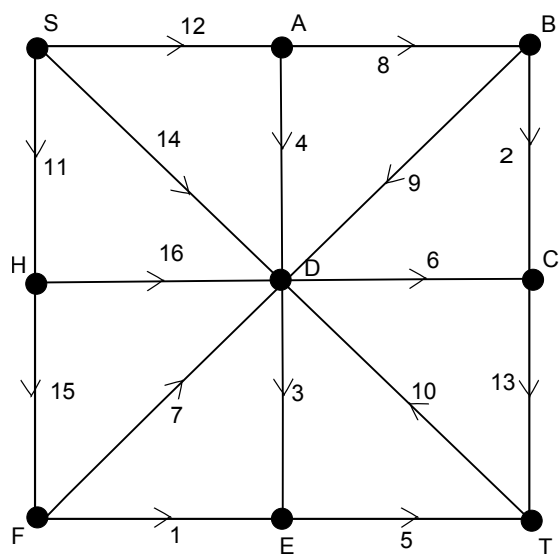
Figure 1: The graph in the question Q1.

Figure 2: The graph in the questions Q2/Q3.