MATH 350: Graph Theory and Combinatorics. Fall 2013.
Due in class on Tuesday, December 3rd.

Assignment #5: Planar graphs.

1. A graph G is *outerplanar* if it can be drawn in the plane so that every vertex is incident with the infinite region. Show that a graph G is outerplanar if and only if G has no K_4 or $K_{2,3}$ minor. (*Hint:* Add a vertex appropriately and use Kuratowski's theorem.)

2. Let G be a simple 2-connected graph drawn in the plane so that all vertices are incident with the infinite region. Suppose that every bounded region of G has length 3. Let k be the number of vertices of degree 2 in G, and let r be the number of regions of G sharing no edges with the infinite region. Show that $k = r + 2$ if $|V(G)| > 3$.

3. Let G be a graph drawn in the plane. Suppose that there exists a vertex v so that v belongs to the boundary of every region. Show that

$$\alpha(G) \geq \frac{1}{2}(|V(G)| - 1).$$

4. Let G be a graph drawn in the plane. Show that G is bipartite if and only if every region of the drawing has even length.

5. Let G be drawn in the plane so that

- the boundary of the infinite region is some cycle C
- every other region has boundary a cycle of length 3
- every vertex of G not in C has even degree.

Show that $\chi(G) \leq 3$. (*Hint:* Use induction. For the induction step consider two cases: whether some two non-consecutive vertices of C are adjacent. In the second case, delete an edge of C and apply the induction hypothesis.)