MATH 350: Graph Theory and Combinatorics. Fall 2013.
Due in class on Thursday, November 21st.

Assignment #4: Matchings and coloring.

1. Let G be a graph and $Z \subseteq V(G)$. Show that the following are equivalent:

(i) G has a matching covering Z, and

(ii) for every $X \subseteq V(G)$ there are at most $|X|$ odd components C of $G \backslash X$ such that $V(C) \subseteq Z$.

2. Let G be a 2-connected 3-regular graph and let e be an edge of G. Show that e belongs to some perfect matching of G.

3. Show that if G is a loopless graph, $k \geq 1$ is an integer and $\chi(G) > k$ then G has a path with k edges.

4. Let G be a loopless graph in which every two odd cycles share a vertex. Show that $\chi(G) \leq 5$.

5. Let G be a loopless graph with $\chi(G) = k$ for some positive integer k. Show that G contains at least k vertices with degree $\geq k - 1$.