MATH 350: Graph Theory and Combinatorics. Fall 2013.
Due in class on Thursday, November 7th.

Assignment #3: Network flows and Ramsey's theorem.

1. Let G be a directed graph and for each edge e let $\phi(e) \geq 0$ be an integer, so that for every vertex v,

$$\sum_{e \in \delta^-(v)} \phi(e) = \sum_{e \in \delta^+(v)} \phi(e)$$

Show there is a list $C_1, ..., C_n$ of directed cycles (possibly with repetition) so that for every edge e of G,

$$|\{i : 1 \leq i \leq n,\ e \in E(C_i)\}| = \phi(e).$$

2. Let s, t be vertices of a digraph G, and let $\phi : E(G) \to \mathbb{R}_+$ be an $s-t$ flow. Show that there is an $s-t$ flow $\psi : E(G) \to \mathbb{Z}_+$ so that

(i) its total value is at least that of ϕ, and

(ii) $|\psi(e) - \phi(e)| < 1$ for every edge e of G.

3. Show that $R(3,4) = 9$.

4. Show that any coloring of edges of K_n with $n \geq 6$ in two colors contains at least $\frac{1}{20}\binom{n}{3}$ monochromatic triangles.

5. Let

$$r_k := R_k(\underbrace{3, 3, \ldots, 3}_{k \text{ times}}).$$

(I.e. r_k is the minimum integer $n > 0$ such that every coloring of edges of K_n in k colors is guaranteed to produce a monochromatic triangle.) Show that

$$r_k \leq k(r_{k-1} - 1) + 2$$

for $k \geq 2$.