MATH 340: Discrete Structures II. Winter 2017.
Due in class on Friday, January 27th.

Assignment #1: Matchings.

1. *Stable matching algorithm.* Apply the Boy Proposal algorithm to find a stable matching given the preference lists below. Are there any other stable matchings?

$$\mathbf{B_1} : G_3 > G_2 > G_1 > G_4 > G_5$$
$$\mathbf{B_2} : G_2 > G_1 > G_3 > G_5 > G_4$$
$$\mathbf{B_3} : G_2 > G_5 > G_4 > G_3 > G_1$$
$$\mathbf{B_4} : G_1 > G_3 > G_4 > G_2 > G_5$$
$$\mathbf{B_5} : G_2 > G_3 > G_1 > G_5 > G_4$$

$$\mathbf{G_1} : B_5 > B_2 > B_1 > B_4 > B_3$$
$$\mathbf{G_2} : B_3 > B_1 > B_4 > B_2 > B_5$$
$$\mathbf{G_3} : B_2 > B_5 > B_4 > B_3 > B_1$$
$$\mathbf{G_4} : B_1 > B_3 > B_4 > B_5 > B_2$$
$$\mathbf{G_5} : B_4 > B_1 > B_5 > B_3 > B_2$$

2. *Stable roommates.* We wish to pair up an even number of students in a student dormitory. Each student has a preference list over every other potential roommate. Give an example to show that a stable matching need not exist.

3. *Edge-coloring.* Let G be a (not necessarily bipartite) graph with maximum degree $\Delta > 0$.

a) Show that $\chi'(G) \leq 2\Delta - 1$.

b) Suppose that G has a perfect matching M such that $G \setminus M$ is bipartite. Determine $\chi'(G)$ in terms of Δ. Justify your answer.

Reminder: $G \setminus M$ is the graph obtained from G by deleting all the edges of M.

4. *Counting matchings.* Let G be a graph with bipartition (A, B) such that $A = \{a_1, a_2, \ldots, a_n\}$, $B = \{b_1, b_2, \ldots, b_{n+1}\}$ and the vertex a_i is adjacent to vertices $b_1, b_2, \ldots, b_{i+1}$ for every $i = 1, 2, \ldots, n$. Show that there are exactly 2^n matchings in G covering A.

5. *Kőnig's theorem.* Let G be a bipartite graph with bipartition (A, B), such that $|A| = |B| = 10$, and every vertex of G has degree at least five. Show that G has a perfect matching.

Hint: Show that if X is a vertex cover of G then either $|X \cap A| \geq 5$ and $|X \cap B| \geq 5$, or $A \subseteq X$, or $B \subseteq X$.

6. *Matching markets.* Consider a matching market with with four buyers (A, B, C, D) and four sellers (X, Y, Z, W), where the valuations of the buyers are listed in the following table.

	X	Y	Z	W
A	6	4	6	6
B	6	5	7	2
C	4	1	7	5
D	3	1	6	3

Use the method seen in class to find a set of market clearing prices.