

MATH 240: Discrete structures I. Fall 2011

Assignment #3: Number theory. Due Monday, October 31st.

- 1.** *Prime factorization.* Show that  $\sqrt[3]{3}$  is irrational.
- 2.** *Euclid's Algorithm.* Use the Euclid's Algorithm to find each of the following.
  - (a)  $\gcd(1230, 96)$ ;
  - (b)  $\gcd(34, 411)$ .
- 3.** *Congruences.* Evaluate the following:
  - (a)  $36^{1620} \pmod{17}$ ;
  - (b)  $36^{1620} \pmod{30}$ .
- 4.** *Modular equations.* Solve the following equations. Show your work.
  - (a)  $5x + 1 \equiv 0 \pmod{13}$ ;
  - (b)  $17x - 5 \equiv 0 \pmod{211}$ ;
  - (c)  $x^2 - 3x + 2 \equiv 0 \pmod{17}$ .
- 5.** *Proofs.*
  - (a) Show that for all integers  $a, b$  and  $k$  we have

$$\gcd(a, b) = \gcd(b, a - kb).$$

- (b) Show that for all positive integers  $m$  and  $n$

$$\gcd(2^m - 1, 2^n - 1) = 2^{\gcd(m, n)} - 1.$$

(*Hint:* Use induction on  $m + n$  and the result of (a).)