Classifying bundles with fiber $K(\pi, 1)$

In the Postnikov system for a map $f : X \to B$ there are commutative diagrams

\[ \begin{array}{ccc}
X^n & \xrightarrow{p^n} & B \\
\downarrow{u^n} & & \downarrow{p^{n-1}} \\
X & \xrightarrow{q^n} & B \\
\downarrow{u_{n-1}} & & \downarrow{p_{n-1}} \\
X^{n-1} & \xrightarrow{} & B
\end{array} \]

where the $u^n$ are $n$-connected and the $p^n$ and $q^n$ are $n$-covers for $n \geq 0$. If $F$ is the fiber of $f$ and $F^n$ the fiber of $p^n$, then we have diagrams

\[ \begin{array}{ccc}
F & \xrightarrow{} & F^n \\
\downarrow{F^{n-1}} & & \\
F^{n-1}
\end{array} \]

which are a Postnikov system for $F$. Looking at the commutative diagrams above, we see that the fiber of $q^n$ is the fiber of $F^n \to F^{n-1}$ which is $K(\pi_n(F), n)$. Assuming $f$ is a minimal fibration, the $q^n$ will also be minimal fibrations and hence bundles with fiber $K(\pi_n(F), n)$. So, for $n \geq 2$, the situation is as we indicated at CT 2011. The classifying spaces are homotopy colimits of diagrams of $K(\pi_n, n + 1)$'s over the fundamental groupoid.

The case $n = 1$, is different, however, for then the fiber is $K(\pi_1(F), 1)$, which, in general, is not a simplicial abelian group etc. If we assume $F$ is connected, then $X^0 = B$ and $q^1 : X^1 \to B$ is a bundle with fiber $K(\pi_1(F), 1)$, so we need to know how to classify these. In the literature this problem is either ignored, leading to mistakes, or swept under the rug by assuming $\pi_1(F)$ is abelian. In the talk, we give a complete, general solution to the problem. This is joint work with Andr Joyal.