The Relative Pure-Entire Factorization for Geometric Morphisms

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Abstract

A locale A in a topos $\mathcal{E}$ is said to be a *Stone locale* if it is a compact and zero-dimensional locale. An equivalent description says that A is the locale of ideals of a Boolean algebra in $\mathcal{E}$. A geometric morphism $\varphi: \mathcal{F} \to \mathcal{E}$ is called *entire* (respect. *pure*) if φ is localic and defined by a Stone locale (respect. if $\varphi_*(2_\mathcal{F}) \cong (2_\mathcal{E})$, where $2 = 1 + 1$). In [P.T.Johnstone, *Factorization Theorems for Geometric Morphisms II, Categorical aspects of Topology and Analysis*, Springer, **LNM 915** (1982) 216-233] it is shown that every geometric morphism φ admits a unique factorization $\varphi \cong \psi \cdot \pi$ where ψ is entire and π is pure.

Suppose now that there is a base topos $\mathcal{S}$ over which the toposes are defined and that we only consider geometric morphisms "over $\mathcal{S}$". Also suppose that instead of $2 = 1 + 1$ in the above, we take the object $\Omega_\mathcal{S}$ of truth-values in the topos $\mathcal{S}$. The question that we answer here is the following: under what conditions on $\varphi: \mathcal{F} \to \mathcal{E}$ (over $\mathcal{S}$) does one obtain a relativized version of the pure-entire factorization mentioned above. In the process of answering it in [M. Bunge, J. Funk, M. Jibladze, T. Streicher, *Relative Stone Locales*, in preparation], we encounter several interesting versions of well-known notions, constructive versions of classically known results, and new problems.