# Do Free Distribution Algebras Exist?

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**Abstract**

If $\mathcal{S}$ is an elementary topos and $\Omega_{\mathcal{S}}$ is its subobjects classifier, then the adjoint pair $F \dashv U$ given by $\Omega_{\mathcal{S}}{}^{(-)} \dashv \Omega_{\mathcal{S}}^{(-)}: \mathcal{S}^{op} \to \mathcal{S}$ is tripleable (R.Paré, *Colimits in Topoi*, *Bulletin of the AMS* **80**(1974) 556-561). Moreover, there exists an equivalence between $\mathcal{S}^{op}$ and the category of *complete atomic Heyting algebras* in $\mathcal{S}$ (over $\mathcal{S}$, the latter equipped withy the forgetful functor and its left adjoint – the *free* complete atomic Heyting algebra functor).

In [M.Bunge, J. Funk, M. Jibladze, T. Streicher, *Distribution Algebras*, to appear in *Advances in Mathematics***156**(2000)] we prove a *relative* version of this result with an interesting interpretation in terms of distributions and their algebraically duals. This is done by replacing $\mathcal{S}$ by a topos $\mathcal{E}$ bounded over $\mathcal{S}$, and by replacing $\mathcal{S}^{op}$ by the category of $\mathcal{S}$-valued distributions on $\mathcal{E}$ in the sense of [F.W. Lawvere, *Extensive and Intensive Quantities*, Lectures at Aarhus University Workshop, 1983]. However, we are seemingly forced to make a hypothesis on $\mathcal{E}$ as a topos over $\mathcal{S}$ for the tripleableness to hold. The tripleableness question is in fact only dependent on the existence of a left adjoint to the forgetful functor from the category of *distribution algebras* in $\mathcal{E}$ to $\mathcal{E}$, that is on the existence of *free distribution algebras*. The theorem holds for any topos $\mathcal{E}$ which is an essential localization of a presheaf topos, as well as when the base topos $\mathcal{S}$ is *Set*. The question itself as well as related matters will be discussed in this talk.