CIRCULATORY ANALYSIS:

A NEW PROBABILITY-TYPE CATEGORY FOR APPLICATIONS

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When we observe a sample $\{x_1, x_2, x_3, ...\}$ of a stationary stochastic process, X_t, to find a sample distribution of X_t, we just count the points. If we count the arrows (say, going up arrows – going down arrows), we also get a deep (sample) characteristic of our process. By stationarity - the process winds up over itself - this characteristic is of a cyclical nature. The cycles exhibited by X describe its particular stochastic temporal symmetry properties.

An index of this type, called the **circulation**, C, with a view towards economic applications, was proposed by W. McCausland in his article 'Time reversibility of stationary regular finite-state Markov chains', *Journal of Econometrics*, 136, 303-318, 2007.

We extended the circulation index to more general settings, and developed a circulatory analysis, based on the cumulative circulation function, IC, that parrallels and, in a way, generalizes the probability theory. This circulation theory measures and manipulates the degrees of temporal asymmetry - when the process has different properties whether taken forward or backward in time.

Because all the definitions and results are naturally defined by the setting, a category theoretical approach to the logical foundations of this theory seems appropriate. This is what we try to do in this talk: starting with basic simple counting examples, we proceed to the theory with categorical flavour.

The proposed circulatory analysis has many applications in natural sciences when we analyse the dynamics of stochastic phenomena. For example, many economic studies find evidence of temporal asymmetry: the fuel price mark-up has the tendency to grow fast and diminish slowly, unemployment rates have similar behaviour. Another example is the 'steepness effect' (e.g. in real business cycles), whereby decreases in various measures of economic activity are faster than increases. We finish our talk with these and other examples. Further research directions are also proposed.

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