The 2-localization of a Quillen's model category

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We refer to Quillen's notion of a category $\mathcal{C}$ furnished with a *model structure* $\{\mathcal{W},\, \mathcal{F},\, co\mathcal{F}\}$ (weak equivalences, fibrations, cofibrations), and the construction of the localisation $\mathcal{C}[\mathcal{W}^{-1}]$ at $\mathcal{W}$ as the homotopy category $\mathcal{H}o(\mathcal{C},\, \mathcal{W})$, that is, the quotient by the congruence determined by homotopies in the sets $\mathcal{C}(X,\, Y)$ of morphisms of $\mathcal{C}$.

In this talk I will construct the 2-localisation $\mathcal{C}[\mathcal{W}^{-e}]$ at $\mathcal{W}$ as the homotopy 2-category $\widetilde{\mathcal{H}o}(\mathcal{C},\, \mathcal{W})$, a 2-category with the same arrows of $\mathcal{C}$, and where the homotopies determine the 2-cells, instead of a congruence.

A novel feature is the introduction of a generalisation of cylinder objects which allows the development of Quillen's theory of homotopies and the construction of the homotopy 2-category for an arbitrary category $\mathcal{C}$ and a single arbitrary class $\mathcal{W} \subset \mathcal{C}$. There is a 2-functor $\widetilde{\mathcal{H}o}(\mathcal{C},\, \mathcal{W}) \longrightarrow \mathcal{C}[\mathcal{W}^{-e}]$, which is an isomorphism if $\mathcal{W}$ is *split generated* (an arrow f is split if there exists g such that $fg = id$ or $gf = id$) and satisfies the *3 for 2 property.*

When $\mathcal{W}$ is the class of weak equivalences of a model category, and $\mathcal{C}$ is the category of fibrant-cofibrant objects, taking the set of connected components of the hom categories we obtain Quillen's results.

This is a particular case of a joint work with Martin Szyld and Emilia Descotte, where we have developed a 2-dimensional version of a model category and its homotopy category. It corresponds to the case where the model bicategory is the trivial model bicategory determined by a model category.