(Marks)

1. Evaluate the following integrals.

(5) (a) $\displaystyle\int \frac{x+3}{\sqrt{x^2+6x+5}}\,dx$

(5) (b) $\displaystyle\int_0^{\pi/2} \sin(2x)\,\sin(3x)\,dx$

(5) (c) $\displaystyle\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx$

(5) (d) $\displaystyle\int \frac{e^{8/x}}{x^2(2+e^{8/x})}\,dx$

(5) (e) $\displaystyle\int \frac{2x-7}{(x^2+2x+10)(x+1)}\,dx$

(5) (f) $\displaystyle\int \frac{\sqrt{x^2-1}}{x^3}\,dx$

(5) (g) $\displaystyle\int \frac{2}{(1-x)^2}\,\ln\left(\frac{1+x}{1-x}\right)\,dx$

2. Evaluate the following limits.

(3) (a) $\displaystyle\lim_{x\to 1^+} \left(\ln(x^5-1) - \ln(\sqrt{x}-1)\right)$

(3) (b) $\displaystyle\lim_{x\to 0} (\arcsin x\,\cot x)$

(3) (c) $\displaystyle\lim_{x\to\infty} \left(1-\frac{3}{x^2}\right)^{x^2}$

3. Evaluate the following improper integrals.

(4) (a) $\displaystyle\int_1^{\infty} \frac{3}{(2x-1)(x+1)}\,dx$

(4) (b) $\displaystyle\int_0^{4} \frac{1}{(x-3)^2}\,dx$

(4) 4. In this problem you are given the curve defined by $y=\frac{4}{5}x^{5/4}$, $0\le x\le 9$. Find the length of the curve.

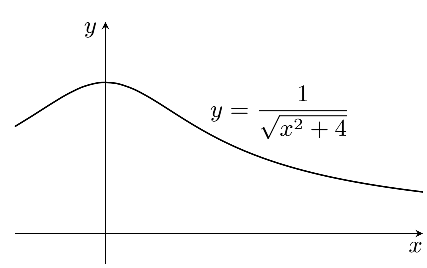
(7) 5. Let $\mathcal{R}$ be the region bounded by $y=\dfrac{1}{\sqrt{x^2+4}}$ and the x-axis between $x=0$ and $x=2$.

(a) Find the volume of the solid obtained by rotating $\mathcal{R}$ about the y-axis.

(b) Set up, but **do not evaluate**, the integrals for the volume of the solid obtained by rotating $\mathcal{R}$ about:

i. the x-axis

ii. the line $y=2$



(4) 6. Solve the differential equation $(4+x^2)^2\,y' = -2\pi x(1+y^2)$ with $y(0)=\dfrac{1}{\sqrt{3}}$. Express y as a function of x.

7. Determine whether each sequence $\{a_n\}$ converges or diverges. If a sequence converges, find what it converges to. Justify your answers.

(2) (a) $a_n = \dfrac{3}{2^n} + \cos(e^{-n})$

(2) (b) $a_n = \dfrac{n!}{3^n}$

(Marks)

(2) 8. Let $\sum_{n=1}^{\infty} a_n$ be the series whose nth partial sum is $s_n = 2 + (-1)^n 2^{-n}$.

(a) Evaluate $\sum_{n=1}^{\infty} a_n$.

(b) Find a_n for $n \geq 2$.

9. Determine whether the following series are convergent or divergent.

(3) (a) $\sum_{n=1}^{\infty} ne^{-n^2}$

(3) (b) $\sum_{n=1}^{\infty} \frac{n}{2n^2 - \cos n}$

(3) (c) $\sum_{n=1}^{\infty} \frac{2^{2n-1} - 1}{n + 3^n}$

10. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

(3) (a) $\sum_{n=0}^{\infty} \frac{3n\,(-3)^n}{(n+3)!}$

(3) (b) $\sum_{n=1}^{\infty} (-1)^n (2 - \arctan n)^{n/2}$

(3) (c) $\sum_{n=1}^{\infty} (-1)^n \ln\left(\frac{n+1}{n}\right)$

(4) 11. For the function $f(x) = \ln(x)$, find the Taylor series around $x = 1$. Write the first five terms of the series explicitly, and express the series in appropriate sigma notation. What is the radius of convergence of the series?

(5) 12. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (3x-2)^n}{8^n (3n+2)^2}$

ANSWERS:

1. (a) $\sqrt{x^2+6x+5} + C$; **(b)** 2/5; **(c)** $\frac{2}{3}\cos^3(\sqrt{x}) - 2\cos(\sqrt{x}) + C$; **(d)** $-\frac{1}{8}\ln|2 + e^{8/x}| + C$; **(e)** $\frac{1}{2}\ln|x^2+2x+10| + \frac{2}{3}\arctan\big(\frac{1}{3}(x+1)\big) - \ln|x+1| + C$; **(f)** $\frac{1}{2}\sec^{-1}(x) - \frac{\sqrt{x^2-1}}{2x^2} + C$; **(g)** $\frac{1+x}{1-x}\ln\left|\frac{1+x}{1-x}\right| - \frac{1+x}{1-x} + C$. **2. (a)** $\ln 10$; **(b)** 1; **(c)** e^{-3}. **3. (a)** $2\ln 2$; **(b)** DIV. **4.** 232/15. **5. (a)** $\int_0^2 \frac{2\pi x}{\sqrt{x^2+4}}\,dx = 4(\sqrt{2}-1)\pi$; **(b - i)** $\int_0^2 \frac{\pi}{x^2+4}\,dx$; **(b - ii)** $\int_0^2 \pi\left[2^2 - \left(2 - \frac{1}{\sqrt{x^2+4}}\right)^2\right]dx$. **6.** $y = \tan\left(\frac{\pi}{x^2+4} - \frac{\pi}{12}\right)$. **7. (a)** 1; **(b)** ∞. **8. (a)** 2; **(b)** $3(-\frac{1}{2})^n$. **9. (a)** C by IT or Ratio; **(b)** D by the LCT with $\sum 1/n$; **(c)** D by the LCT with $\sum(\frac{4}{3})^n$. **10. (a)** AC by the Ratio Test; **(b)** AC by the Root Test; **(c)** CC by AST (use telescoping series to check that AC fails). **11.** $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n}$; $R = 1$. **12.** $[-2,\ 10/3]$.