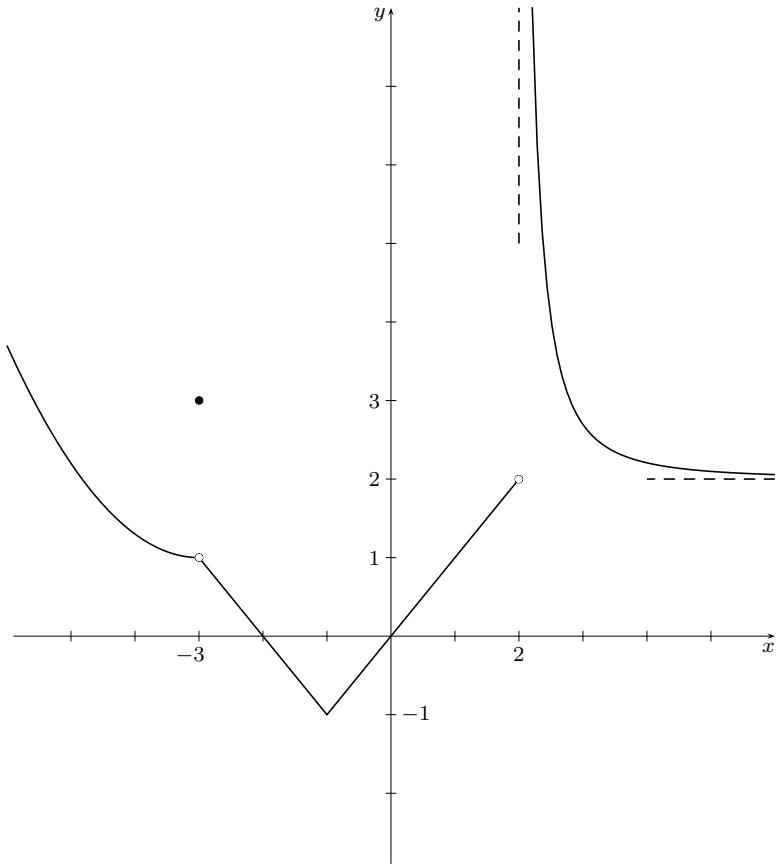
(Marks)

(5) 1. Use the graph of the function $f(x)$ below to determine the following. Use ∞, $-\infty$, or DNE where appropriate.



(a) $\lim_{x\to-\infty} f(x)$

(b) $\lim_{x\to+\infty} f(x)$

(c) $\lim_{x\to-3} f(x)$

(d) $\lim_{x\to2^-} f(x)$

(e) $\lim_{x\to2} f(x)$

(f) $\lim_{x\to-1} f(x)$

(g) $\lim_{x\to0^+} f(x)$

(h) $f(-3)$

(i) List the x-value(s) at which the function $f(x)$ is continuous but not differentiable.

(10) 2. Evaluate the following:

(a) $\lim_{x\to2} \dfrac{x^2+2x-8}{x^4-16}$

(b) $\lim_{t\to\frac{\pi}{3}^-} \dfrac{\sin t}{1-2\cos t}$

(c) $\lim_{x\to4} \dfrac{4-x}{\sqrt{x+5}-3}$

(d) $\lim_{x\to-\infty} \sqrt{\dfrac{2+9x}{5+4x}}$

(e) $\lim_{x\to2} \dfrac{x\ln x - \ln(x^2)}{x-2}$

(4) 3. Given $f(x) = \begin{cases} |x-2| & \text{if } x \le 1 \\ \sqrt{x-1} & \text{if } 1 < x < 5 \\ \dfrac{2x}{10-x} & \text{if } x > 5 \end{cases}$

find all values of x where $f(x)$ is not continuous, and justify your answers. Give the type of discontinuity at each value.

(4) 4. (a) Use the limit definition of the derivative of a function to find the derivative of $f(x) = \dfrac{2x}{7-x}$.

(b) Check your answer to part (a) by using the derivative rules.

(Marks)

(15) 5. Find $\frac{dy}{dx}$ for each of the following:

(a) $y = (3x^2 - 9)\sec^4(5x + 3)$

(b) $y = \ln\left(\frac{(x^2-9)^4}{x^3\sqrt{x+7}}\right)$

(c) $y = \sin\left[(2x-1)^3\right] + \cos^3(2x-1)$

(d) $y = (\sin x)^{\ln x}$

(e) $y = e^{\sqrt{x^2+3}}$

(3) 6. Find an equation of the tangent line to the curve $y = (1 + e^{-x})^4$ at $x = 0$.

(4) 7. Given $f(x) = \frac{x^2+1}{(x+1)^2}$

(a) Find $f'(x)$ and simplify.

(b) Determine whether $f(x)$ has a relative (local) maximum or minimum at $x = 1$.
You may use the fact that $f''(x) = \frac{-4(x-2)}{(x+1)^4}$.

(5) 8. Given the curve $x^2 - xy + y^2 = 9$,

(a) Show that $y' = \frac{y-2x}{2y-x}$.

(b) Find the coordinates of both points on the curve where the tangent line is horizontal.

(8) 9. Given $f(x) = \frac{x^3-8}{x^3+8}$ and $f'(x) = \frac{48x^2}{(x^3+8)^2}$ and $f''(x) = \frac{-192x(x^3-4)}{(x^3+8)^3}$,
Sketch the graph of $f(x)$ clearly showing all (if any) asymptotes, intercepts, local (relative) extrema, and points of inflection.

(3) 10. Find the absolute extrema of $f(x) = x^{1/3} - x^{2/3}$ on $[-1, 1]$.

(3) 11. Let $f(x) = x^2e^x$. Find the intervals where $f(x)$ is increasing or decreasing. (Do not sketch the graph.)

(4) 12. Let θ (in radians) be an acute angle in a right angled triangle and let x and y respectively be the lengths of the sides adjacent to and opposite to θ. Suppose that x and y vary with time. At the instant when $x = 2$ and is increasing at 4 units/second, $y = 2$ and is decreasing at 1 unit/second. How fast is θ changing at this time?

(4) 13. Michael has 28 m of fencing to enclose two separate turtle pens. One pen will be a rectangle three times as long as it is wide, and the other pen will be a square. For the comfort of the turtles, the width of the rectangular pen should be at least 1 m and at most 3 m. Find the maximum and minimum total areas of the pens.

(4) 14. Give an expression for (or exact value of) the following limits:

(a) $\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$

(b) $\lim_{x\to 0^-} \frac{|x|}{x}$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(e^{\frac{2i}{n}}\right) \frac{2}{n}$

(Marks)

(d) $\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta}$

(3) 15. Given the function $f(x) = \sqrt{100 - x^2}$ defined on $[-6, 8]$, verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

(6) 16. Evaluate the following:

(a) $\displaystyle\int_1^2 \frac{(2y+1)^2}{y}\, dy$

(b) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec\theta + \tan\theta}{\cos\theta}\, d\theta$

(c) $\displaystyle\int \sqrt{x}(1 - x + x^2)\, dx$

(3) 17. Use differentiation to verify that $\displaystyle\int \frac{4}{3(4-3x^2)^{3/2}}\, dx = \frac{x}{3\sqrt{4-3x^2}} + C$

(3) 18. Find $z(t)$ given that:

$$\frac{d^2z}{dt^2} = 2\sin t - 4\cos t + e^t \quad \text{and} \quad \left.\frac{dz}{dt}\right|_{t=0} = 3 \quad \text{and} \quad z(0) = -5$$

(4) 19. Let S be the region bounded by $f(x) = \dfrac{3+x}{x}$ and the x-axis between $x = 1$ and $x = 9$.

(a) Approximate the area of S by finding the Riemann sum with four equal subintervals and taking midpoints as sample points.

(b) What is the exact area of S?

(3) 20. Evaluate $\displaystyle\int_0^5 |x - 2|\, dx$ by interpreting it in terms of area.

(2) 21. Use the fundamental theorem of calculus to find a function f and a number a such that

$$8 + \int_a^x \frac{f(t)}{t\sqrt{t}}\, dt = 4\ln x$$

(Marks)

Answers

1.(a) ∞ (b) 2 (c) 1 (d) 2 (e) DNE (f) -1 (g) 0 (h) 3 (i) -1

2.(a) $\frac{3}{16}$ (b) $-\infty$ (c) -6 (d) $\frac{3}{2}$ (e) $\ln 2$

3. jump: $x = 1$, removable: $x = 5$, infinite: $x = 10$ 4.(a) $\dfrac{14}{(7-x)^2}$ (b) $\dfrac{14}{(7-x)^2}$

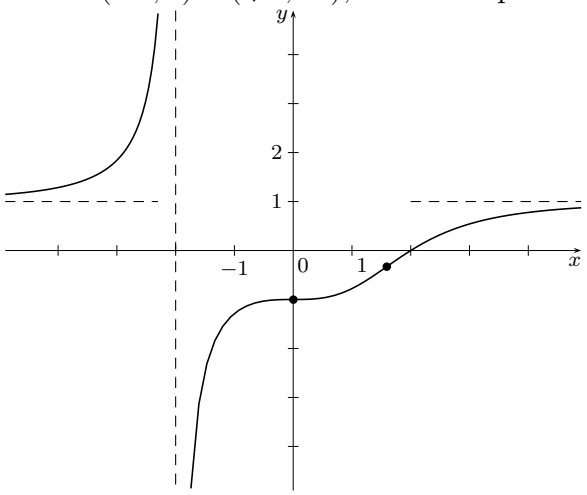
5.(a) $20(3x^2-9)\sec^4(5x+3)\tan(5x+3) + 6x\sec^4(5x+3)$ (b) $\dfrac{8x}{x^2-9} - \dfrac{3}{x} - \dfrac{1}{2(x+7)}$

(c) $6(2x-1)^2\cos\left((2x-1)^3\right) - 6\cos^2(2x-1)\sin(2x-1)$ (d) $(\sin x)^{\ln x}\left(\ln x\cot x + \dfrac{\ln(\sin x)}{x}\right)$

(e) $\dfrac{xe^{\sqrt{x^2+3}}}{\sqrt{x^2+3}}$ 6. $y = -32x + 16$ 7.(a) $\dfrac{2(x-1)}{(x+1)^3}$ (b) $f''(1) > 0 \Longrightarrow$ loc. min. at $x = 1$

8.(a) differentiate implicitly (b) $(\sqrt{3}, 2\sqrt{3})$ and $(-\sqrt{3}, -2\sqrt{3})$

9. domain: $\Re\backslash\{-2\}$, y-int: $(0,-1)$ and x-int: $(2,0)$, HA: $y = 1$ and VA: $x = -2$, crit num: $x = 0$, increasing: $(-\infty,-2)\cup(-2,0)\cup(0,\infty)$, decreasing: nowhere, concave up: $(-\infty,-2)\cup(0,\sqrt[3]{4})$, concave down: $(-2,0)\cup(\sqrt[3]{4},\infty)$, inflection points: $(0,-1)$ and $(\sqrt[3]{4},-\frac{1}{3})$



10. abs min: $(-1,-2)$, abs max: $(\frac{1}{8},\frac{1}{4})$ 11. increasing: $(-\infty,-2)\cup(0,\infty)$, decreasing: $(-2,0)$

12. $-\frac{5}{4}$ rad/sec 13. max area: $28\,\text{m}^2$, min area: $21\,\text{m}^2$

14.(a) $\cos x$ (b) -1 (c) $e^2 - 1$ (d) 1 15. $c = \sqrt{2}$

16.(a) $10 + \ln 2$ (b) $1 - \sqrt{2} + \sqrt{3}$ (c) $\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + \frac{2}{7}x^{7/2} + C$

17. differentiate RHS and simplify to obtain integrand 18. $z = -2\sin t + 4\cos t + e^t + 4t - 10$

19.(a) $A \approx \frac{57}{4}$ (b) $A = 3\ln 9 + 8$ 20. $\frac{13}{2}$ 21. $f(x) = 4\sqrt{x}$ and $a = e^2$