(4) 1. What is the equation of the tangent line to the graph of $y = \arctan(\sqrt{2x+1})$ at the point where $x = 0$?

(32) 2. Evaluate the integrals.

(a) $\displaystyle\int_0^5 x\sqrt{9-x}\,dx$ (b) $\displaystyle\int \sec^6(t)\sqrt{\tan t}\ dt$ (c) $\displaystyle\int_0^{1/10} \arcsin(5x)\,dx$

(d) $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$ (e) $\displaystyle\int \frac{3x^3+3x^2-12x+20}{x^4+4x^2}\,dx$ (f) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

(g) $\displaystyle\int e^{2x}\cos x\,dx$

(8) 3. Evaluate the improper integrals.

(a) $\displaystyle\int_0^\infty \frac{\arctan x}{1+x^2}\,dx$ (b) $\displaystyle\int_0^1 x\ln x\,dx$

(9) 4. Evaluate the limits.

(a) $\displaystyle\lim_{x\to+\infty}\left(1+\frac{4}{x-2}\right)^x$ (b) $\displaystyle\lim_{x\to1^+}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$ (c) $\displaystyle\lim_{x\to0^+} e^{-1/x}\ln x$

(2) 5. Let $f(x)$ be a continuous function with a continuous derivative for $0 \le x < \infty$, satisfying

- $f(0) = 2$,
- $0 \le f(x) \le 100$ (for all x in $[0,\infty)$),
- $\displaystyle\int_0^\infty e^{-x} f(x)\,dx = 15$.

Use this information to determine the value of $\displaystyle\int_0^\infty e^{-x} f'(x)\,dx$.

(10) 6. Let $\mathcal{R}$ be the region above $y = x$, below $y = e^x$, and between $x = 0$ and $x = 4$.

(a) Compute the area of the region $\mathcal{R}$.

(b) Set up the integrals required to compute the volume of the solid obtained by rotating $\mathcal{R}$ about (i) the x-axis, and (ii) the line $x = 4$.

(c) Compute the volume of the solid obtained by rotating $\mathcal{R}$ about the y-axis.

(4) 7. Solve the differential equation: $yy'\sqrt{x^2+1} = x$; $y(0) = 1$.

(3) 8. Does the sequence $\left\{\sqrt{n^4+n^3}-n^2\right\}$ converge? If so, find its limit as $n\to\infty$. Justify your answer.

(2) 9. Mark each statement as TRUE (if it is necessarily true) or FALSE (otherwise). Justify your answers.

(a) If $a_n > 0$ and $\sum a_n$ converges, then $\sum \sin a_n$ converges.

(b) If $a_n > 0$ and $\sum \sin a_n$ converges, then $\sum a_n$ converges.

(9) 10. Determine whether each of the following series converges or diverges. State the tests you use, and verify that the conditions for using them are satisfied.

(a) $\displaystyle\sum_{n=0}^\infty \frac{\sec^2 n}{\sqrt{n+1}}$ (b) $\displaystyle\sum_{n=0}^\infty \frac{n^{3n}}{(3n)!}$ (c) $\displaystyle\sum_{n=0}^\infty \frac{\sqrt[3]{n^4+3}}{3n^2+7}$

(6) 11. Label each series as absolutely convergent, conditionally convergent, or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=2}^\infty (-1)^n\frac{\ln n}{n}$ (b) $\displaystyle\sum_{n=0}^\infty (-1)^n e^{-\sqrt{n}}$

(2) 12. If a power series $\displaystyle\sum_{n=0}^\infty a_n(x-3)^n$ converges at $x = 5$, must it also converge at $x = 0$; must it also converge at $x = 4$? Briefly justify your answers.

(4) 13. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n 3^n (2x-1)^n}{\sqrt[3]{n}}$.

(5) 14. Find the Taylor series of $f(x) = x\ln x$ centered at $x = 1$. For what values of x does the Taylor series converge?

NYB Exam Answers — May 2008

1. $y = \frac{1}{2}x + \frac{\pi}{4}$
2. (a) $\frac{148}{5}$ (b) $\frac{2}{3}(\tan t)^{3/2} + \frac{4}{7}(\tan t)^{7/2} + \frac{2}{11}(\tan t)^{11/2} + C$
   (c) $\frac{\pi}{60} + \frac{\sqrt{3}}{10} - \frac{1}{5}$ (d) $\arcsin(e^x) + C$ (e) $-3\ln|x| - \frac{5}{x} + 3\ln(x^2+4) - \arctan\frac{x}{2} + C$
   (f) $2\arcsin(x/2) - \frac{1}{2}x\sqrt{4-x^2} + C$ (g) $\frac{1}{5}e^{2x}(\sin x + 2\cos x) + C$
3. (a) $\pi^2/8$ (b) $-1/4$
4. (a) e^4 (b) $-1/2$ (c) 0
5. 13
6. (a) $e^4 - 9$
   (b) (i) $\pi\int_0^4 (e^{2x} - x^2)\,dx$ (ii) $2\pi\int_0^4 (4-x)(e^x - x)\,dx$
   (c) $2\pi(3e^4 - 61/3)$
7. $y = \sqrt{2\sqrt{x^2+1} - 1}$
8. $\to \infty$ (*i.e.* diverges)
9. (a): True (use LCT on $\sum|\sin(a_n)|$) (b): False (*e.g.* $\sum \sin(\pi n)$)
10. (a) Diverges (CT) (b) Converges (RT) (c) Diverges (LCT)
11. (a) CC (CT & AST) (b) AC ($\int$T)
12. Need not C at $x = 0$; must C at $x = 4$.
13. $R = 1/6$; I of C: $\left(\frac{1}{3}, \frac{2}{3}\right]$
14. Taylor series: $(x-1) + \sum_{n=2}^{\infty} \frac{(-1)^n (x-1)^n}{(n-1)n}$; I of C: $[0, 2]$