1. Differentiate $y = 2x \operatorname{arcsec} x - \sqrt{x^2 - 1}$ with respect to x, and simplify your answer.

2. Evaluate the integrals.

   (a) $\displaystyle\int_0^{1/4} \frac{\arccos(2x)}{\sqrt{1-4x^2}}\,dx$ (b) $\displaystyle\int \frac{(x+2)}{\sqrt{2x+1}}\,dx$ (c) $\displaystyle\int x \arctan x\,dx$

   (d) $\displaystyle\int \tan^3(\tfrac{x}{2})\,dx$ (e) $\displaystyle\int_0^{\pi/4} \sin^3 2x \cos^2 2x\,dx$ (f) $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}\,dx$

   (g) $\displaystyle\int \frac{3x^2 - 2x + 9}{(x-1)(x^2+4)}\,dx$

3. Evaluate the improper integrals.

   (a) $\displaystyle\int_1^{\infty} \frac{1}{x^2+2x}\,dx$ (b) $\displaystyle\int_{-1}^{2} \frac{(x+1)}{[x(x+2)]^{4/3}}\,dx$

4. Evaluate the limits.

   (a) $\displaystyle\lim_{x\to+\infty} \left(\frac{x+2}{x+3}\right)^x$ (b) $\displaystyle\lim_{x\to 1} \frac{x - \mathrm{e}^{x-1}}{(x-1)^2}$ (c) $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{x\cos x}\right)$

5. Let $\mathcal{S}$ be the region bounded by the graphs of $x = y - y^2$ and $x = 0$

   (a) Compute the area of the region $\mathcal{S}$.

   (b) Find the volume of the solid obtained when this region is rotated about the x-axis.

   (c) Find the volume of the solid obtained when this region is rotated about the y-axis.

6. Let $\mathcal{R}$ be the region bounded by the graphs of

   $y = x \sin x$, $y = 0$, $x = 0$, and $x = 2$.

   (a) Set up the integrals required to compute the volume of the solid obtained by rotating $\mathcal{R}$ about (i) the x-axis, and (ii) the y-axis.

   (b) Evaluate **one** of the integrals from part (a).

7. Solve the differential equation: $(x^2+1)y' = y$; $y(1) = 2$.

8. Does the sequence $\left\{\dfrac{3(n-1)!}{5(n+1)!}\right\}$ converge? If so, find its limit as $n \to \infty$. Justify your answer.

9. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \left(\arccos\left(\tfrac{1}{n+1}\right) - \arccos\left(\tfrac{1}{n}\right)\right)$ converges or diverges; if it converges, find the sum. Justify your answer.

10. Determine whether each of the following series converges or diverges. State the tests you use, and verify that the conditions for using them are satisfied.

    (a) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n+2}{2n+1}\right)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\mathrm{e}^{1/n}}{n^2}$

11. Label each series as absolutely convergent, conditionally convergent, or divergent. Justify your answers.

    (a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n(n+1)}{(n+2)(n+3)}$ (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(n!)^2}{(2n)!}$

12. Find the radius and interval of convergence of the power series

    $$\sum_{n=0}^{\infty} \frac{3^n\,(x-2)^n}{n^3+1}$$

13. Find the Taylor series of $f(x) = \dfrac{1}{x}$ centered at 1.