1. (8 points) Evaluate the following limits:
   (a) $\displaystyle\lim_{x\to -3} \frac{x^2+3x}{2x^2+5x-3}$
   (b) $\displaystyle\lim_{x\to 2} \frac{\sqrt{2x+5}-3}{x^2-4}$
   (c) $\displaystyle\lim_{x\to -\infty} \frac{5-8x^3}{(3x^2-2)(2x+3)}$
   (d) $\displaystyle\lim_{x\to 0} \frac{\tan(3x)}{x}$

2. (5 points) Sketch the graph of

$$f(x)=\begin{cases} x^2-4 & \text{if } x<-2 \\ \sqrt{2-x} & \text{if } -2\leqslant x<2 \\ x-2 & \text{if } x\geqslant 2 \end{cases}$$

   (a) For what value(s) of x (if any) is $f(x)$ not continuous? Justify your answer.
   (b) For what value(s) of x (if any) is $f(x)$ not differentiable? Justify your answer.

3. (3 points) Find the value of k that will make

$$f(x)=\begin{cases} \dfrac{4-x^2}{x^3+8} & \text{if } x<-2 \\ x+k & \text{if } x\geqslant -2 \end{cases}$$

   continuous at $x=-2$.

4. (18 points) For each of the following functions, find the derivative dy/dx. **Do not simplify your answers.**
   (a) $y=e^{-5x}\cos 8x+\tan^3(2x^2+5)$
   (b) $x^2y^3=x\sin y+4$
   (c) $y=\dfrac{3}{4x^2}-6\sqrt[3]{x^4}+3^{x^2}-\log_5(\sqrt{x})+\pi^3$
   (d) $y=(x^2-4)^{3x+2}$
   (e) $y=\sqrt{x^2+\sqrt{16-x^4}}$
   (f) $y=\ln\dfrac{(3x^2+2)^5\sin 2x}{e^{4x}\sqrt{x^3+2}}$

5. (8 points) Find dy/dx for each of the following functions and **simplify your answers**.
   (a) $y=\left(\dfrac{1-x}{x^2+x-1}\right)^7$
   (b) $y=\sqrt{4x^2-1}\,(x^2-2)^4$

6. (4 points) Given $f(x)=\sqrt{3x-2}$, find $f'(x)$ using the limit definition of the derivative of a function.

7. (4 points) Find the point(s) on the ellipse $3x^2+y^2=12$ where the tangent line has a slope of 1.

8. (4 points) Find the equation of the tangent line to the curve $y=\dfrac{x}{x+4}$ at $x=4$.

9. (6 points) Find the absolute extrema for the following functions on the given intervals:
   (a) $f(x)=x^3-4x^2-3x$ on the interval $[-1,2]$
   (b) $f(x)=\cos^2 x+4\sin x$ on the interval $[0,2\pi]$

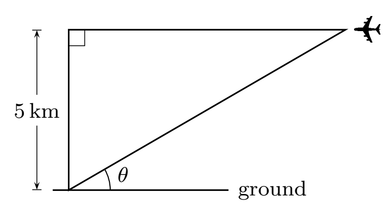
10. (8 points) Let $f(x)=\dfrac{25x^{2/5}}{x+6}$. Then

$$f'(x)=\frac{15(4-x)}{x^{3/5}(x+6)^2} \qquad f''(x)=\frac{24(x+1)(x-9)}{x^{8/5}(x+6)^3}$$

   Also, $f(4)\approx 4.35$ and $f(9)\approx 4.01$.
   Sketch the graph of $f(x)$ identifying **clearly** all intercepts, asymptotes, local extrema and points of inflection. Show all work.

11. (5 points) A piece of wire 14 m long is cut into two pieces. One piece is bent to form a square and the other is bent to form a rectangle that is three times as long as it is wide. How should the wire be cut so that the sum of the two areas is:
   (a) a minimum?
   (b) a maximum?

12. (5 points) An airplane is flying at an altitude of 5 km towards a point directly above an observer on the ground. The speed of the airplane is 600 km per hour. At the instant that the observer's angle of elevation to the plane is $\theta=\pi/6$, find:
   (a) the rate at which angle θ is changing,
   (b) the rate at which the distance between the observer and the plane is changing.



13. (9 points) Evaluate the following integrals:
   (a) $\displaystyle\int\left(\frac{7}{y^4}-\sqrt{y^3}+\frac{5}{y}-e^3\right)dy$
   (b) $\displaystyle\int \sec\theta\tan\theta\csc\theta\,d\theta$
   (c) $\displaystyle\int_1^2 \frac{(2x-1)^2}{x}\,dx$

14. (4 points) Find $f(x)$ knowing that $f''(x)=e^x+\sin x-2$, $f'(0)=3$ and $f(0)=5$.

15. (3 points) Let $f(x)=(1-x)(x-3)$.
   (a) Sketch and shade in the region S bounded by $f(x)$ and the x-axis.
   (b) Find the area of S.

16. (6 points) Are the following statements *True* or *False*? You must **briefly** justify your answer.

(a) $f(x) = \dfrac{x^3 - 4x}{x - 2}$ has a vertical asymptote at $x = 2$.

(b) $\dfrac{d}{dx}\left(\displaystyle\int_x^e \ln(1 + t^2)\,dt\right) = \ln(1 + x^2)$

(c) If $f(x)$ is continuous at $x = a$, then it must be differentiable at $x = a$.

(d) If $\displaystyle\int f(x)\,dx = x^2 \ln x + C$, then $f(x) = x + 2x \ln x$.

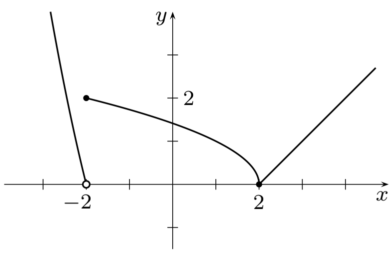
(e) $\displaystyle\int_\pi^\pi \sqrt{\tan x}\,dx = 0$

(f) $\displaystyle\int_{-1}^{2} x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(-1 + \frac{3i}{n}\right)^2 \frac{3}{n}$

Answers

1. (a) $\frac{3}{7}$ (b) $\frac{1}{12}$ (c) $-\frac{4}{3}$ (d) 3

2.



(a) -2 (b) -2, 2

3. $\frac{7}{3}$

4. (a) $-5e^{-5x}\cos 8x - 8e^{-5x}\sin 8x + 12x\tan^2(2x^2+5)\sec^2(2x^2+5)$

(b) $-\dfrac{3}{2x^3} - 8\sqrt[3]{x} + 3^{x^2}\cdot 2x\ln 3 - \dfrac{1}{2x\ln 5}$ (c) $\dfrac{\sin y - 2xy^3}{3x^2y^2 - x\cos y}$

(d) $(x^2-4)^{3x+2}\left[3\ln(x^2-4) + \dfrac{2x(3x+2)}{x^2-4}\right]$

(e) $\frac{1}{2}(x^2 + \sqrt{16 - x^4}\,)^{-1/2}\left[2x + \frac{1}{2}(16 - x^4)^{-1/2}(-4x^3)\right]$

(f) $\dfrac{30x}{3x^2+2} + \dfrac{2\cos 2x}{\sin 2x} - 4 - \dfrac{3x^2}{2(x^3+2)}$

5. (a) $\dfrac{7x(x-2)(1-x)^6}{(x^2+x-1)^8}$ (b) $\dfrac{4x(9x^2-4)(x^2-2)^3}{\sqrt{4x^2-1}}$

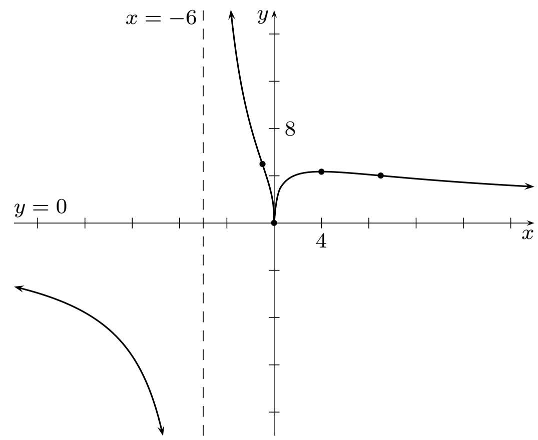
6. $\displaystyle\lim_{h\to 0} \frac{\sqrt{3(x+h)-2} - \sqrt{3x-2}}{h} = \frac{3}{2\sqrt{3x-2}}$

7. $(1,-3)$, $(-1,3)$ **8.** $y - \frac{1}{2} = \frac{1}{16}(x-4)$ or $y = \frac{1}{16}x + \frac{1}{4}$

9. (a) Abs. max. $f(-\frac{1}{3}) = \frac{14}{27}$, abs. min. $f(2) = -14$

(b) Abs. max. $f(\pi/2) = 4$, abs. min. $f(3\pi/2) = -4$

10. x-int. 0; y-int. 0; VA $x = -6$; HA $y = 0$;
Loc. max. $f(4) = \frac{5}{2}\sqrt[5]{16} \approx 4.35$; Loc. min. $f(0) = 0$;
IP $(-1,5)$, $(9, \frac{5}{3}\sqrt[5]{81}) \approx (9, 4.01)$

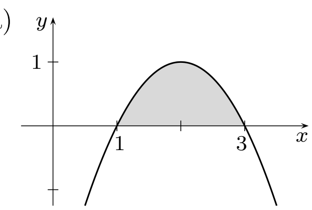


11. (a) 6 m for the square, 8 m for the rectangle
(b) Use all of the wire for the square

12. (a) Increasing at a rate of 30 rad/h (b) Decreasing at a rate of $300\sqrt{3}$ km/h

13. (a) $-\dfrac{7}{3y^3} - \frac{2}{5}y^2\sqrt{y} + 5\ln|y| - e^3 y + C$ (b) $\tan\theta + C$
(c) $2 + \ln 2$ **14.** $e^x - \sin x - x^2 + 3x + 4$

15. (a)



(b) $\frac{4}{3}$

16. (a) False: f has a removable discontinuity at $x = 2$
(b) False: the derivative equals $-\ln(1 + x^2)$
(c) False: f could have a corner or cusp at $x = a$
(d) True: $d(x^2\ln x + C)/dx = x + 2x\ln x$
(e) True: property of the definite integral
(f) True: the right side is a limit of Riemann sums involving a regular partition of $[-1, 2]$ with $\Delta x = 3/n$ and right endpoints $x_i = -1 + 3i/n$, for $i = 1, \ldots, n$