(4) 1. Find $\dfrac{dy}{dx}$ if $y = (\arctan x)^2 + x\,\arctan x - \operatorname{arcsec}\sqrt{x}$. Do not simplify your answer.

(6) 2. Calculate the following limits: (a) $\displaystyle\lim_{x\to\infty} x\left(e^{1/x} - 1\right)$ (b) $\displaystyle\lim_{x\to 0^+} (1+2x)^{\frac{1}{3x}}$

(24) 3. Evaluate the following integrals.

(a) $\displaystyle\int \left(\frac{1}{2\sqrt{x}} + \frac{2}{x}\right) 3^{\sqrt{x}+2\ln x}\, dx$

(b) $\displaystyle\int \frac{\sec^3(\ln x)\,\tan^3(\ln x)}{x}\, dx$

(c) $\displaystyle\int \frac{x+2}{\sqrt{2x-1}}\, dx$

(d) $\displaystyle\int \frac{x^2+3}{x^3+2x}\, dx$

(e) $\displaystyle\int e^{2x}\sin 4x\, dx$

(f) $\displaystyle\int_3^{3\sqrt{2}} \frac{1}{x^3\sqrt{x^2-9}}\, dx$

(6) 4. Determine whether the improper integrals converge or diverge; if an integral converges, give its exact value.

(a) $\displaystyle\int_1^2 \frac{3}{\sqrt{4-x^2}}\, dx$ (b) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

(3) 5. Compute the exact area of the region bounded by the curves $y = 4 - x^2$ and $y = x^2 + 2$ between $x = 0$ and $x = 2$. Sketch the region and show an element of area.

(6) 6. (a) Sketch the region R bounded by $y = 4 - x^2$ and $y = x^2 + 2$ on the interval $[1, 2]$.

(b) Set up, *BUT DO NOT EVALUATE*, the integral required to find the volume of the solid obtained when R is revolved

i. about the line $x = 1$

ii. about the x axis.

Show an element of volume in each case.

(5) 7. Solve the differential equation: $\dfrac{\sqrt{1-x^2}}{y}\dfrac{dy}{dx} + x = 0$; $y(0) = 1$. (An explicit solution, please)

(6) 8. Determine, with justification, whether the sequence converges or diverges. If a sequence converges, give the value to which it converges.

(a) $\left\{\dfrac{\ln^3 n}{n}\right\}$ (b) $\left\{\dfrac{e^{3/n}}{3n}\right\}$ (c) $\left\{(-1)^n\cos^2 n\pi\right\}$

(6) 9. Determine whether the series converges or diverges. If a series converges, give its sum.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\operatorname{arcsec} n - \operatorname{arcsec}(n+1)\right)$ (b) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{(-1)^n}{e^n} + \frac{2}{3^n}\right)$

(12) 10. Classify each series as convergent or divergent. State the test you use, and verify the conditions for using the test are satisfied.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(4 + \frac{2}{n}\right)$ (b) $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt[n]{3} - 1\right)^n$ (c) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n^2}\right)$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2\,2^n}{(2n)!}$

(8) 11. Classify each series as absolutely convergent, conditionally convergent, or divergent. Justify your conclusions.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin^2\left(\frac{n\pi}{2}\right)}{n^3}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\,\ln n}$

(4) 12. Determine the interval of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1}(x-1)^n}{(n+1)!}$

13. (a) Use a known power series to determine the sum of the following series:

(1) i. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots$

(1) ii. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\,(\pi)^{2n}}{(6)^{2n}\,(2n)!}$

(2) (b) Use a known power series to find a power series for $f(x) = x\,e^{-2x}$. What is its interval of convergence?

(6) 14. For the function $f(x) = \ln(x+1)$

(a) find the first five non-zero terms of the Maclaurin series for $f(x)$

(b) find the n^{th} term and write the series in sigma notation

(c) determine its radius of convergence.