Math 201-NYA-05, Calculus 1, Winter 2006

1. Evaluate the limits. Use the symbols $-\infty$ or $+\infty$ where appropriate.

[2] (a) $\lim\limits_{x\to 3}\left(\dfrac{x^3-x^2-6x}{x^2+2x-15}\right)$

[2] (b) $\lim\limits_{x\to 2}\left(\dfrac{2-\sqrt{x+2}}{x^2-4}\right)$

[2] (c) $\lim\limits_{x\to -\infty}\left(\dfrac{\sqrt{x^2+1}}{2x}\right)$

[2] (d) $\lim\limits_{x\to +\infty}\left(\dfrac{3x^3-2x^2+6}{5x^2+x-8}\right)$

[2] (e) $\lim\limits_{x\to \frac{\pi}{2}^+}\left(\dfrac{\tan x}{x}\right)$

[4] 2. Use the limit definition of the derivative to find $f'(x)$ if $f(x)=\sqrt{3x+2}$.

[4] 3. Find the values of the constants c and d that make $f(x)$ continuous on $\mathbb{R}$.

$$f(x)=\begin{cases} c\,x+10 & \text{if } x\le -2 \\ d|x| & \text{if } -2<x\le 2 \\ c(x-2)^2+6 & \text{if } x>2 \end{cases}$$

4. Find the derivative of each of the following. Do not simplify.

[3] (a) $y=\dfrac{2x}{x^{3/4}}-\dfrac{4x^{2/3}}{x}+4^{3/2}-(3x)^2$

[3] (b) $y=\sqrt{x}\,\tan(x)$

[3] (c) $y=\dfrac{\ln x}{1+\sin x}$

[3] (d) $y=\sqrt{\sec\left(\dfrac{x+1}{x-1}\right)}$

[3] (e) $y=(x^3+2x)^{\sec x}$

[3] 5. Use logarithmic differentiation to find the derivative of

$$y=\frac{(x+1)^{2/3}(3x-1)^{1/2}(5x+1)^{1/5}}{(2x+1)^{1/2}(x-1)^{1/3}}$$

6. For the curve whose equation is

$$27x^3+y^3=18xy$$

[3] (a) Find an expression for the derivative y' in terms of x and y.

[3] (b) Find the equation of the tangent line at $(1, 3)$.

7. The position of a particle in motion along a straight line is given by the equation

$$s = t^3 + 3t^2 - 24t,$$

where t is measured in seconds and s in meters.

[2] (a) Find the average velocity over the period from $t = 1$ to $t = 5$.

[2] (b) Find an expression for the instantaneous velocity at time t.

[2] (c) Find when the particle is stationary (that is, when its velocity is zero.)

[2] (d) Find the acceleration of the particle when it is stationary.

[5] 8. If a balloon is being inflated so that its surface area is increasing at a rate of 1 cm^2 per second, find the rate at which the diameter is increasing when the diameter is 20 cm.
The surface area, A, of a sphere of radius r is given by $A = 4\pi r^2$.

[5] 9. Find the absolute maximum and absolute minimum values of the function

$$f(x) = (x^2 - 1)^{2/3}$$

on the interval $[0, 3]$.

[12] 10. For the function

$$g(x) = x + \frac{4}{x^2}$$

(a) Find the x-intercept(s).

(b) Find $\lim_{x\to-\infty} g(x)$ and $\lim_{x\to+\infty} g(x)$

(c) Find the vertical asymptote(s).

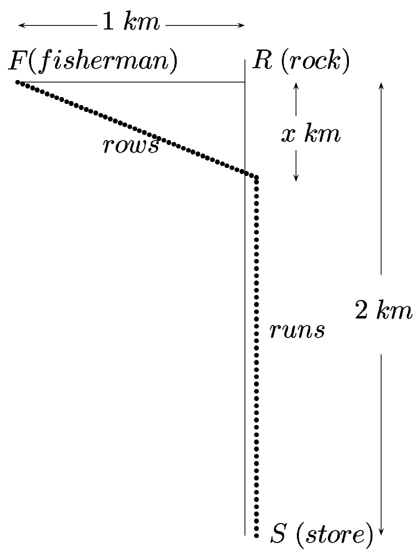
(d) Find the coordinates of all relative extrema.

(e) Find the intervals on which the function is increasing and decreasing.

(f) Find any inflection points and the intervals on which the function is concave up and concave down.

(g) Sketch a graph of the function on the making sure that your graph illustrates all these features. Use the separate page of graph paper for your sketch.

[6] 11. A fisherman is in a rowboat on a lake with a shoreline which is straight . A large blue rock is at the point on the shoreline closest to him, and it is exactly 1 km away from him. He has to reach a shop 2 km further along the shoreline from the rock. He can row at 5 km/hr, and he can run at 13 km/hr. When he heads for the shore, how far from the rock should he land in order to get to the shop as quickly as possible ?
(Suggestion: let x km be the distance from the rock to the point that he lands.)



12. Find the general antiderivative

[3] (a) $\displaystyle\int \frac{4\cos x}{\sin^2 x}\,dx$

[3] (b) $\displaystyle\int x^{2/3}(x^{-4/3} - 3)\,dx$

[4] 13. Find the function $f(x)$, that satisfies the given conditions
$f''(x) = 2x \qquad f'(0) = -3,\quad$ and $f(0) = 2$.

[3] 14. Use the Fundamental Theorem of Calculus to find the following derivative.

$$\frac{d}{dx}\int_0^{x^2} \left(e^{-t^2} + 1\right) dt$$

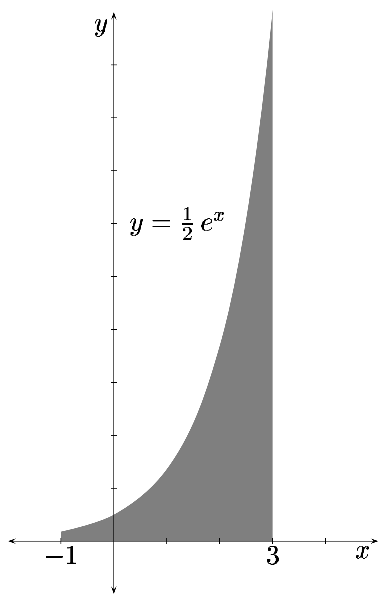
15. Evaluate the following definite integrals.

[3] (a) $\displaystyle\int_1^4 \frac{x-3}{x}\,dx$

[3] (b) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} (2\sin x - \cos x)\,dx$

[3] 16. Use the definite integral to determine the area of the region bounded by

$y = \frac{1}{2}e^x, \quad x = -1, \qquad x = 3$ and the x-axis.



Answers to Calculus 1 exam, Winter 2006

1. (a) $\frac{15}{8}$ (b) $-\frac{1}{16}$ (c) $-\frac{1}{2}$ (d) $+\infty$ (e) $-\infty$;

2. $f'(x) = \lim_{h\to 0} \frac{3}{\sqrt{3x+3h+2}+\sqrt{3x+2}} = \frac{3}{2\sqrt{3x+2}}$;

3. $c = 2$ and $d = 3.$;

4. (a) $\frac{1}{2}\,x^{-3/4} + \frac{4}{3}\,x^{-4/3} - 18\,x$ (b) $x^{1/2}\sec^2 x + \frac{1}{2}\,x^{-1/2}\tan(x)$ (c) $y' = \frac{(1+\sin x)\frac{1}{x} - (\ln x)(\cos x)}{(1+\sin x)^2}$

(d) $y' = -\sqrt{\sec\left(\frac{x+1}{x-1}\right)}\cdot\tan\left(\frac{x+1}{x-1}\right)\cdot\frac{1}{(x-1)^2}$ (e) $y' = (x^3+2x)^{\sec x}\left[(\sec x)\left(\frac{3\,x^2+2}{x^3+2\,x}\right) + (\sec x)(\tan x)\ln(x^3+2\,x)\right]$;

5. $y' = \frac{(x+1)^{2/3}(3x-1)^{1/2}(5x+1)^{1/5}}{(2x+1)^{1/2}(x-1)^{1/3}}\left(\frac{2}{3x+3} + \frac{3}{6x-2} + \frac{1}{5x+1} - \frac{1}{2x+1} - \frac{1}{3x-3}\right)$;

6. (a) $y' = \frac{6y - 27x^2}{y^2 - 6x}$ (b) $y = -3x + 6.$;

7. (a) $v_{\text{avr}} = 25$ (m/s) (b) $v(t) = 3t^2 + 6t - 24$

(c) $t = -4$ (seconds) and $t = 2$ (seconds) (d) $a(t) = 18$ (m/s^2) ;

8. $\frac{1}{40\,\pi}$ cm/s ;

9. absolute maximum is 4 and absolute minimum is 0 ;

10. (a) $(-\sqrt[3]{4}, 0)$ (b) $\lim_{x\to-\infty} g(x) = +\infty$ and $\lim_{x\to+\infty} g(x) = -\infty$ (c) $x = 0$ (d) relative minimum at $(2, 3)$

(e) increases on $(-\infty, 0)$ and $(2, +\infty)$ decreases on $(0, 2)$ (f) concave up on $(-\infty, 0)$ and $(0, +\infty)$,

no inflection points

11. $5/12$ km ;

12. (a) $-4\csc x + C$ (b) $3\,x^{1/3} - 9/5\,x^{5/3} + C$;

13. $f(x) = (1/3)x^3 - 3\,x + 2$;

14. $\left(e^{-x^2} + 1\right)2\,x$;

15. (a) $3 - 6\ln 2$ (b) 3 ;

16. $(1/2)\,[e^3 - e^{-1}]$