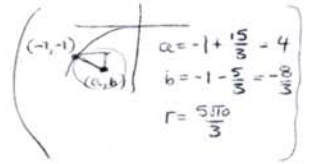


8c) $\vec{u} = \langle 1, 1, 1 \rangle$, $\vec{v} = \langle 1, -2, 1 \rangle$, so $\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = 0$ so $\theta = 90^\circ = \frac{\pi}{2}$
 (d) $\vec{a} = \langle 0, 2, 0 \rangle$, so: $a_T = v' = \frac{4t}{\sqrt{2+4t^2}}$, $a_N = \frac{|\vec{v} \times \vec{a}|}{v} = \frac{\sqrt{8}}{\sqrt{2+4t^2}} = \frac{2}{\sqrt{1+2t^2}}$



9 $K = \frac{6}{\sqrt{1+9}} = \frac{3}{5\sqrt{10}}$ so $r = \frac{5\sqrt{10}}{3}$, so $(x-4)^2 + (y+\frac{8}{3})^2 = \frac{250}{9}$

10 $\frac{\partial z}{\partial x} = -\frac{f_x}{f_z} = -\frac{y+z}{2y^2+x} \Big|_{(1,1,1)} = -\frac{2}{3}$

11 $\frac{\partial z}{\partial x} = f' - g'$, $\frac{\partial z}{\partial y} = af' + ag'$; $\frac{\partial^2 z}{\partial x^2} = f'' - g''$, $\frac{\partial^2 z}{\partial y^2} = a^2 f'' - a^2 g''$ so qed

12 $\vec{\nabla} f = \langle 24x^2 - 12x, 3y^2 - 12y \rangle = \vec{0}$ if $x = 0, \frac{1}{2}$, $y = 0, 4$.

$D = \begin{vmatrix} 48x-12 & 0 \\ 0 & 6y-12 \end{vmatrix} = 72(4x-1)(y-2)$. At $(0,0)$ ($D > 0, f_{xx} < 0$) MAX. At $(\frac{1}{2}, 0)$ and $(0, 4)$ ($D < 0$) Saddle, at $(\frac{1}{2}, 4)$ ($D > 0, f_{xx} > 0$) MIN.

13 $\begin{cases} 1 = 2x\lambda \\ 2 = 2y\lambda \\ 2 = 2z\lambda \\ x^2 + y^2 + z^2 = 3 \end{cases}$ so $y = z = \frac{1}{\lambda}$, $x = \frac{1}{2\lambda} = \frac{1}{2}y$ } so pts are $(-\frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}})$ (min) and $(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$ (max)

14 $R: \begin{cases} 0 \leq \theta \leq \frac{\pi}{2} \\ 0 \leq r \leq 2\sin \theta \end{cases}$ } so integral = $\int_0^{\frac{\pi}{2}} \int_0^{2\sin \theta} r^2 dr d\theta = \frac{8}{3} \int_0^{\frac{\pi}{2}} \sin^3 \theta d\theta = \frac{8}{3} (\frac{1}{3} \cos^3 \theta - \cos \theta) \Big|_0^{\frac{\pi}{2}} = \frac{16}{9}$

15 $V = \int_0^4 \int_0^{y/2} (4-x) dx dy = \int_0^4 \int_{2x}^4 (4-x) dy dx = \int_0^4 \int_0^{y/2} \int_0^{4-x} dz dx dy = \frac{40}{3}$

16 $S =$ the region in the top hemisphere of radius = 4 with the cone ($\varphi = \frac{\pi}{6}$) removed

17 $Mass = \iiint_S \sqrt{x^2 + y^2 + z^2} dV = \int_0^{2\pi} \int_0^{\pi/4} \int_0^3 \rho^3 \sin \varphi d\rho d\varphi d\theta = 2\pi \cdot (1 - \frac{1}{\sqrt{2}}) \cdot \frac{81}{4} (= 37.265)$

