DDB Exam May 2005 Solutions

1) (a) $f = 1+\sum_{n=2}^{\infty} \frac{3^n(x-5)^{n+1}}{(n+1)!}$ (b) $R = +\infty$, = I of C = $\mathbb{R}$ (c) $f^{(7)}(5) = 3^6$

2) (a) $= \int_0^x \left(1-\frac{x^2}{3}+\frac{x^4}{5}-+\cdots\right)dx = x - \frac{x^3}{3^2} + \frac{x^5}{5^2} -+\cdots = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)^2}$. R of C = 1

(b) $= x^2(x-2)^{-1} = -\frac{1}{2}x^2\left(1-\frac{x}{2}\right)^{-1} = -\frac{1}{2}x^2\left[1+\frac{x}{2}+\frac{x^2}{2^2}+\frac{x^3}{2^3}+\cdots\right] = -\sum_{n=0}^{\infty}\frac{x^{n+2}}{2^{n+1}}$; $R = 2$

3) $f(x) = (16+(x-16))^{1/4} = 2\left(1+\frac{x-16}{16}\right)^{1/4} = 2\left[1+\frac{1}{4}\left(\frac{x-16}{16}\right)+\frac{(1)(-3)}{4\cdot 4\cdot 2!}\left(\frac{x-16}{16}\right)^2+\cdots\right]$

so $T_2(x) = 2+\frac{x-16}{32}-\frac{3}{4096}(x-16)^2$; $\sqrt[4]{15} \approx T_2(15) = 2-\frac{1}{32}-\frac{3}{4096} = \frac{8061}{4096}$ $(=1.9680175)$

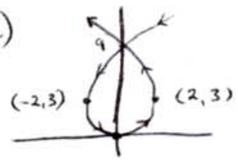
error: $M = \frac{21}{64}15^{-11/4}$, so $|R| \le \frac{M}{6} = 0.000032$ (so correct to 4dp): $\sqrt[4]{15} = 1.96801 \pm 0.000032$

4) a) $\dot{x} = 3-3t^2$, $\dot{y} = 6t$; $\frac{dy}{dx} = \frac{6t}{3-3t^2} = \frac{2t}{1-t^2}$: VT if $t=\pm 1$: $(-2,3)$, $(2,3)$

HT if $t=0$: $(0,0)$ Intercepts: $(0,0)$, $(0, 9)$ $(t=0$ or $\pm\sqrt{3})$

b) $\frac{d^2y}{dx^2} = \frac{2+2t^2}{3(1-t^2)^3}$: PPI at $t=\pm 1$. $\cup$ if $-1<t<1$, $\cap$ if $t<-1$ or $t>+1$.

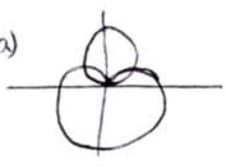
c)



(d) length $= \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{(3-3t^2)^2+(6t)^2}\,dt = 3\int_{-\sqrt{3}}^{\sqrt{3}}(1+t^2)\,dt = 12\sqrt{3} = 20.78$

(e) area $= 2\int_0^{\sqrt{3}}(3t-t^3)\cdot 6t\cdot dt = \frac{72\sqrt{3}}{5} = 24.94$

5) (a)



(b) area $= 2\int_0^{\pi/6}\frac{1}{2}(2\sin\theta)^2\,d\theta + 2\int_{\pi/6}^{\pi/2}\frac{1}{2}(2-2\sin\theta)^2\,d\theta$

$2-2\sin\theta = 2\sin\theta$

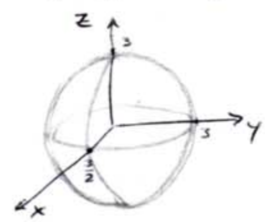
$\Rightarrow \sin\theta = \frac{1}{2}$, $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

(c) length $= \int_{5\pi/6}^{13\pi/6}\sqrt{[2-2\sin\theta]^2+[-2\cos\theta]^2}\,d\theta$

6) (a) [$z=0$: ellipse $4x^2+y^2=9$] Ellipsoid:

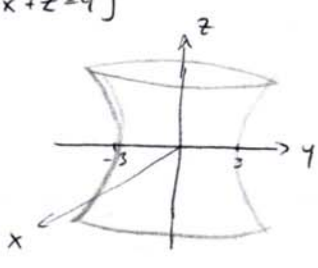
[$x=0$: ellipse $y^2+z^2=9$]

[$y=0$: ellipse $4x^2+z^2=9$]



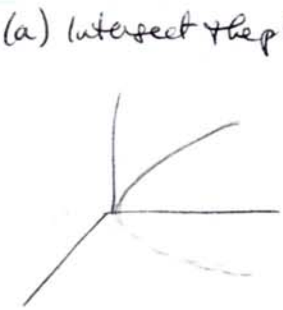
(b) $r^2 - z^2 = 9$

(rotate)



7) $\rho^4 = 2r^2z = 2\rho^3\sin^2\varphi\cos\varphi \rightarrow \rho = 2\sin^2\varphi\cos\varphi$

8) (a) Intersect the plane $z=x$ and the parabolic cylinders $y=x^2$ (and $y=z^2$):



(b) Intersection: $t+t^2+t+1=0 \rightarrow (t+1)^2=0 \rightarrow t=-1 \rightarrow (-1,1,-1)\,P_0$

direction $\vec{v} = \langle 1, 2t, 1\rangle = \langle 1,-2,1\rangle$ at P_0

So the line is $\begin{cases} x = -1+t \\ y = 1-2t \\ z = -1+t \end{cases}$