(Marks)

(9) 1. (a) Sketch the graphs of the circle $r = 2$ and the cardioid $r = 2(1 - \cos\theta)$, on the same axes.

(b) Find the area of the region that lies inside the intersection of these two curves. Indicate this region on your graph.

(c) Find the length of the circumference of the cardioid $r = 2(1 - \cos\theta)$.

(5) 2. Find the Maclaurin series for $f(x) = \dfrac{1}{\sqrt{4-x}}$ and find its radius of convergence. (Hint: You might want to use the Binomial Theorem.)

(8) 3. (a) Find the Maclaurin series representation of $\int_0^x \sin(t^2)\,dt$.

(b) Estimate $\int_0^1 \sin(t^2)\,dt$ with an error less than 0.001. Be sure to justify your answer.

(9) 4. Sketch the following surfaces:

(a) $y^2 - x^2 = z$

(b) $z^2 = 1 + r^2$

(c) $\rho = \cos\varphi$

(10) 5. A curve is defined by $\boldsymbol{r}(t) = 2t\,\boldsymbol{i} + t^2\,\boldsymbol{j} + \frac{1}{3}t^3\,\boldsymbol{k}$

(a) Find the unit tangent and unit normal vectors $\boldsymbol{T}, \boldsymbol{N}$, and the curvature κ.

(b) Find the tangential and normal components of acceleration $a_{\boldsymbol{T}}, a_{\boldsymbol{N}}$.

(Hint: the algebra does work out well if you carefully simplify!)

(6) 6. (a) Is the following function continuous at the origin? Be sure to justify your answer.

$$f(x) = \begin{cases} \dfrac{5xy}{2x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(b) Find $f_x(0,0)$ and $f_y(0,0)$.

(6) 7. The temperature distribution in a fluid is given by $T(x,y,z) = (x-10)^2 + 2y^2 + 3(z-2)^2$.

(a) Determine the rate of temperature change at the point $P_0(9,0,3)$ in the direction $\langle 11, 2, 4\rangle$.

(b) At P_0, what is the greatest rate of temperature change, and in which direction is it greatest?

(6) 8. (a) Find the equation of the tangent plane to the ellipsoid $4x^2 + y^2 + z^2 = 9$ at the point $P_0(-1,1,2)$.

(b) Find the parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $4x^2 + y^2 + z^2 = 9$ at the point $P_0(-1,1,2)$.

(6) 9. (a) If $h = (x+y+z)^2$, $x = s - t$, $y = \cos(s+t)$, and $z = \sin(s+t)$, find $\dfrac{\partial h}{\partial t}$ when $s = -1,\ t = 1$.

(b) Show that $w = f(x + ct)$, f differentiable, c constant, is a solution of the wave equation $\dfrac{\partial^2 w}{\partial t^2} = c^2 \dfrac{\partial^2 w}{\partial x^2}$.

(6) 10. Find and classify the critical points of $f(x,y) = 4xy - x^4 - y^4$.

(7) 11. Use Lagrange Multipliers to find the point (x,y,z) on the sphere $x^2 + y^2 + z^2 = 1$ which is furthest from the point $(1,2,3)$.

(Marks)

12. Evaluate the following.

(5) (a) $\displaystyle\int_0^1 \int_x^1 \sqrt{1+y^2}\; dy\, dx$

(5) (b) $\displaystyle\int_0^{3/\sqrt{2}} \int_x^{\sqrt{9-x^2}} \mathrm{e}^{-(x^2+y^2)}\; dy\, dx$

(6) 13. Let $\mathcal{S}$ be the solid region above ("inside") the the cone $z = \sqrt{x^2+y^2}$, and below ("inside") the sphere $x^2+y^2+z^2 = z$. Sketch the region $\mathcal{S}$, and find its volume.
(I suggest you try spherical coordinates.)

(6) 14. Use the transformation $\{\, x = u - v\,,\ y = 2u - v \,\}$ to evaluate the integral $\displaystyle\iint_{\mathcal{R}} \sqrt{y-x}\; dx\, dy$,

where $\mathcal{R}$ is the parallelogram bounded by the lines
$y = 2x\,,\ \ y = 2x - 2\,,\ \ y = x\,,\ \ y = x + 1\,.$