Marks

1. Evaluate the following integrals:

(5) a) $\int \frac{9}{(x-1)^2(2x+1)}\,dx$

(4) b) $\int \frac{1}{x^2\sqrt{49-x^2}}\,dx$

(4) c) $\int \sqrt{\tan(3x)}\,\sec^4(3x)\,dx$

(5) d) $\int \sin(2x)\cos(3x)\,dx$

(4) e) $\int_0^{1/3} \frac{\arctan(3x)}{1+9x^2}\,dx$

(4) f) $\int_0^5 \frac{x}{\sqrt{3x+1}}\,dx$

(4) g) $\int \frac{3x^2+3x+2}{x^2+1}\,dx$

(5) h) $\int x \operatorname{arcsec} x\,dx$

2. Calculate the following limits:

(4) a) $\lim_{x\to\infty} (e^x + x)^{2/x}$

(3) b) $\lim_{x\to\pi/2} \frac{\sin^2 x - 1}{\cos 2x + \sin x}$

(8) 3. Determine whether the following integrals converge or diverge. If an integral converges, give the exact value.

a) $\int_e^\infty \frac{dx}{x\,(\ln x)^2}$

b) $\int_0^3 \frac{1}{(x-1)^{4/3}}\,dx$

(4) 4. Find the solution of the differential equation:

$$2y\,e^x \frac{dy}{dx} - x = 0 \quad ; \quad y(0) = 2$$

(3) 5. Consider the sequence $\{a_n\} = \left\{\frac{n^2+2}{2n(n+1)}\right\}$

a) Does the sequence converge and if so, to what value?

b) Does the corresponding series $\sum_{n=1}^{\infty} a_n$ converge? (Justify your answer.)

(12) 6. Determine whether the following series converge or diverge. State the test you are using and display a proper solution.

a) $\sum_{n=1}^{\infty} \left(\frac{5n-1}{3n+2} \right)^n$

b) $\sum_{n=1}^{\infty} \left(\frac{3}{2^n} - \frac{1}{n\sqrt{n}} \right)$

c) $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$

d) $\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{\sqrt{n}}$

7. Determine whether the following series are absolutely convergent, conditionally convergent or divergent:

(3) a) $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{n^2\, 3^n}$

(4) b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{3n-1}}$

8. Given the series $\sum_{n=1}^{\infty} \frac{3^{n-1}}{5^{n+1}}$

(2) a) Find a formula for S_n, the n^{th} partial sum of the series.

(1) b) Find the sum of the series.

(5) 9. Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3^n (x-2)^{n+1}}{2n+1}$$

(4) 10. a) Find the first 3 non-zero terms of the Taylor series expansion of $f(x) = \sin(x)$ centered at $x = \pi/2$.

b) Use Sigma notation to write the general form of the series in (a).

(4) 11. a) Sketch the region R bounded by $y^2 = x + 2$ and $y = x$

b) Find the area of R.

(8) 12. Let R be the shaded region bounded by $y = \cos x$ the $y-$axis and $y = \sin x$.

FIGURE

a) **Set up the integral(s)** needed to find the volume of the solid of revolution that results when R is rotated about:

i) the $x-$axis

ii) the $y-$axis

b) Evaluate **one** of the above integrals (i) or (ii). (**Not Both**)