(Marks)

(12) 1. Consider the curve given by the equations

$$x = 3t - t^3 \quad \text{and} \quad y = 3t^2$$

(a) Find the t-values and the coordinates of the x and y intercepts.

(b) Find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$. Use these to sketch the graph of the curve for $-2 \le t < 2$. Carefully indicate horizontal and vertical tangents, and intervals where the curve is concave up and where it is concave down. Label relevant points with their t values as well as their xy coordinates.

(c) The curve forms a loop in this region; find the area of that loop.

(d) Find the length of the arc that forms the loop.
(Hint: the algebra does work out well if you carefully simplify!)

(7) 2. (a) Sketch the graphs of $r = 2 - 2\sin\theta$ and $r = 2\sin\theta$, on the same axes. Find all points of intersection.

(b) Find the area of the region that lies inside the intersection of these two curves.

(9) 3. (a) Find the Maclaurin series for $\int_0^x e^{-t^4}\, dt$.

(b) What is the interval of convergence for this power series?

(c) Use the answer to 3(a) to approximate $\int_0^{1/2} e^{-t^4}\, dt$ to within an error of $\pm 10^{-4}$. (Justify your approximation.)

(6) 4. What is the Maclaurin series for the function $f(x) = \frac{1}{(1-3x)^3}$? (Hint: Binomial theorem)
Use Taylor's inequality to estimate the error in using the second degree Maclaurin polynomial $T_2(x)$ to approximate $f(x)$ for $|x| \le 0.01$.

(6) 5. Sketch a graph of the surface $\frac{x^2}{4} - \frac{y^2}{4} - z^2 = 1$. Find the equation of the tangent plane at the point $P_0\,(3, 1, 1)$.

(6) 6. Suppose you are on a surface $z = 100 - \frac{1}{2}x^2 - \frac{1}{4}y^2$ at the point $P\,(2, 2, 97)$.

(a) In which direction should you go to climb up the surface as steeply as possible?

(b) If you go in that direction, how steep is your climb (*i.e.* what is the slope)?
(Bonus: what is the angle of your ascent, measured relative to the horizontal?)

(10) 7. A curve is defined by $\boldsymbol{r}(t) = \langle t\,,\, t^2\,,\, \frac{2}{3}t^3 \rangle$. Find the unit tangent and unit normal vectors $\boldsymbol{T}$, $\boldsymbol{N}$, and the curvature κ. Find the tangential and normal components of acceleration $a_{\boldsymbol{T}}, a_{\boldsymbol{N}}$.

(8) 8. (a) If $z = f(x^2 - y^2)$, f a differentiable function, show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

(b) If $z = z(x, y)$ is implicitly defined by the equation $x\sin(yz) + y\cos(xz) = xyz$, find $\frac{\partial z}{\partial y}$.

(6) 9. Find and classify the critical points of $f(x, y) = x^2 - 4xy + y^3 + 4y$.

(8) 10. Use Lagrange Multipliers to find the minimum value of $f(x, y, z) = 4x^2 + y^2 + 6z^2$ on the plane $x + 2y + 3z = 23$.

(Marks)

11. Evaluate the following.

(5) (a) $\int_0^1 \int_y^1 \cos(x^2)\, dx\, dy$

(5) (b) $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2)\; dz\, dy\, dx$

(6) 12. Let $\mathcal{S}$ be the solid region above the xy plane, inside the cylinder $x^2+y^2=4$, and below the cone $z^2=x^2+y^2$. Evaluate $\iiint_{\mathcal{S}} \frac{1}{x^2+y^2+z^2}\, dV$. (I suggest you try spherical coordinates.)

(6) 13. Use the transformation $\left\{ x = \frac{u+v}{2},\ y = \frac{u-v}{2} \right\}$ to evaluate the integral $\iint_{\mathcal{R}} e^{x-y}\; dA$, where $\mathcal{R}$ is the diamond-shaped region bounded by the lines $x+y=2$, $x+y=4$, $x-y=0$, $x-y=2$.