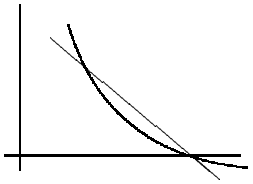
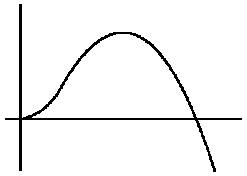
(Marks)

(3) 1. Find the area of the region between the curves $y = \dfrac{2}{x} - 1$ and $y = 2 - x$.



(5) 2. Let R be the region (as shown) bounded by $y = \sin(x^2)$, $y = 0$, $x = 0$, and $x = \sqrt{\pi}$.



(a) Find the exact volume of the solid that results from revolving R about the y-axis.

(b) Set up the integral required to find the volume of the solid that results from revolving R about the x-axis.
Do not actually evaluate the integral.

(4) 3. If $y = \dfrac{\operatorname{arcsec}\sqrt{x}}{e^{2x}}$ find y'. (Do not simplify.)

(30) 4. Evaluate the following integrals:

(a) $\displaystyle\int e^{2x}\sin x\,dx$ (b) $\displaystyle\int \frac{2}{t^2}\left(1-\frac{1}{t}\right)^2 dt$ (c) $\displaystyle\int \frac{dx}{\sqrt{1-16x^2}}$

(d) $\displaystyle\int \sin^2 3x\cos^2 3x\,dx$ (e) $\displaystyle\int \frac{8x^2-3x-4}{(4x-1)(x^2+1)}\,dx$ (f) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x^2+1}}$

(9) 5. Calculate the following limits

(a) $\displaystyle\lim_{x\to 1}\frac{x-e^{x-1}}{(x-1)^2}$ (b) $\displaystyle\lim_{x\to\infty}\left(\frac{x-1}{x+2}\right)^x$

(c) $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$

(8) 6. Determine whether these improper integrals converge or diverge: if an integral converges, give the exact value of the integral.

(a) $\displaystyle\int_0^1 \frac{x+1}{(x^2+2x)^{5/4}}\,dx$ (b) $\displaystyle\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

(5) 7. Find the solution of the following differential equation.
$(x+1)e^y\dfrac{dy}{dx} = x\,,\ x \ge 0\,,\ y(0) = 1$

(3) 8. For the sequence $\{a_k\} = \left\{(-1)^k\cos\left(\dfrac{1}{k}\right)\right\}$, determine whether or not it is convergent. (Justify your answer.)

(3) 9. Calculate (if possible) the sum of the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2n^2+2n}$

(12) 10. Classify each of the following series as convergent or divergent. (Briefly justify your conclusions.)

(a) $\displaystyle\sum_{n=0}^{\infty}\frac{(n!)^2}{(2n)!}$ (b) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n^2}\right)$

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(c) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n^2}\right)^n$ (d) $\displaystyle\sum_{n=0}^{\infty}\frac{\sqrt{n^3-1}}{n^2+1}$

(8) 11. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Briefly justify your conclusions.)

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\,\frac{\arctan n}{n^2+1}$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\cos\left(\frac{1}{n}\right)$

(5) 12. Determine the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{3^{n-1}\,(x+1)^n}{n\sqrt{n+1}}$.

(5) 13. For the function $f(x)=e^{2x}$

(a) find the first five terms of the Maclaurin series for $f(x)$;

(b) find the n^{th} term, and express the series in Σ notation.

(c) What is the radius of convergence for this series?