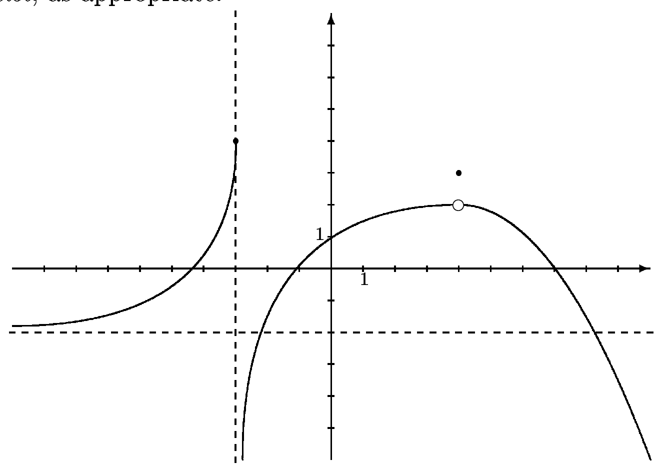
(3) 1. (a) Refer to the accompanying graph of $y = f(x)$ in order to evaluate the following. Use the terms ∞, $-\infty$, and *does not exist*, as appropriate.



(i) $\lim_{x \to -\infty} f(x) =$ (ii) $\lim_{x \to -3^+} f(x) =$ (iii) $\lim_{x \to 4^-} f(x) =$

(2) (b) Show that $f(x)$ is discontinuous at $x = 4$.

(1) (c) Classify the discontinuity at $x = 4$.

(12) 2. Determine each of the following limits. Use the terms ∞, $-\infty$, and *does not exist*, as appropriate. Include adequate justification and use correct mathematical notation.

(a) $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$

(b) $\lim_{x \to 2^+} \frac{x^2 - 5x + 6}{x^2 - 4x + 4}$

(c) $\lim_{x \to 3} \frac{(x-8)^2 - 25}{3 - x}$

(d) $\lim_{x \to -\infty} \frac{3 + 9x^2}{1 + 3x}$

(4) 3. Let $g(t) = \begin{cases} 3t + 2 & \text{if } t \leq 1 \\ 5 & \text{if } 1 < t \leq 3 \\ 3t^2 - 1 & \text{if } t > 3 \end{cases}$.

Determine all points of discontinuity of the function $g(t)$. Justify your answers.

(4) 4. The position of a particle moving in a straight line is given by

$$s = t^4 - 8t^3 + 16t^2 + 2$$

where t is measured in seconds and s in meters.

(a) Show that the particle is at rest when $t = 4$.

(b) Find the acceleration at $t = 4$.

(4) 5. (a) Use the limit definition of derivative to find $f'(x)$ for $f(x) = \frac{2}{x+1}$.

(20) (b) Find the derivatives of the following functions. Do not simplify.

(i) $y = 5x^{10} + \frac{8}{\sqrt[4]{x}} + e^{\csc x}$

(ii) $y = x^5 \ln(x^3 + 1)$

(iii) $y = \dfrac{x^3 + 2x}{x^7 - 3}$

(iv) $y = \cos^2(Ax + B)$

(v) $y = (1 + x)^{\sin x}$

(4) 6. Find the second derivative of the function $f(x) = \sec x \tan x$.

(3) 7. Compute $\dfrac{dy}{dx}$ for $x + \ln(xy) = 2$. Find an equation for the tangent line to the curve at the point $(1, e)$.

(4) 8. At what values of x does $f(x) = (5x - 2)^4(1 - x)^6$ have a horizontal tangent line.

(4) 9. The infield of a 400 metre track consists of a rectangle with semicircular ends.

To what dimensions should the track be built in order to maximize the area of the rectangle?

(3) 10. Find the extreme values (*i.e.*, the absolute maximum and absolute minimum) of

$$p(x) = 4x^3 + 6x^2 - 72x + 13$$

on the interval $[-2, 3]$.

(4) 11. The area of a square is decreasing at a rate of 10 m^2/h. How fast is the diagonal of the square decreasing when the length of a side is 5 metres.

(10) 12. Find all x- and y-intercepts, vertical and horizontal asymptotes, local extrema, point(s) of inflection, intervals of increasing/decreasing and intervals of concavity. Then sketch the graph of

$$f(x) = \frac{2 + x - x^2}{(x-1)^2} \; ; \; f'(x) = \frac{x - 5}{(x-1)^3} \; ; \; f''(x) = \frac{2(7 - x)}{(x-1)^4}.$$

(3) 13. Evaluate the following integral by interpreting it in terms of area: $\displaystyle\int_{-5}^{0} \sqrt{25 - x^2}\, dx$

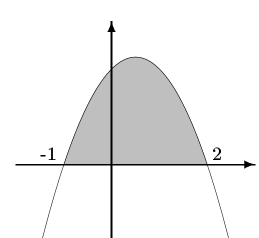
(9) 14. Find:

(a) $\displaystyle\int \frac{x^7}{3} + \frac{4}{x^2} - 5\sqrt[3]{x^2} + 6 \, dx$

(b) $\displaystyle\int \frac{2\sin x - 5}{\cos^2 x} \, dx$

(c) $\displaystyle\int \frac{e^x}{3} + \frac{5}{x} - 8x^{1/2} \, dx$

(3) 15. Find the area between the curve $y = 2 + x - x^2$ and the x-axis.



(3) 16. $f''(x) = 3\cos x + 2\sin x$, $f(0) = 3$ and $f'(0) = 2$. Solve for $f(x)$.