(Marks)

(8) 1. (a) Find the Maclaurin series for $\displaystyle\int_0^x \frac{1-e^{-t^2}}{t^2}\,dt$.

(b) What is the interval of convergence for this power series?

(c) Use the answer to 1(a) to approximate $\displaystyle\int_0^{1/2} \frac{1-e^{-t^2}}{t^2}\,dt$ correctly to 4 decimal places. (Justify your approximation.)

(7) 2. Given $f(x) = \dfrac{1}{1-x}$:

(a) Find the Maclaurin series for $f(x)$ and for $f'(x)$. What is the radius of convergence for these power series?

(b) Use your answer to 2(a) to find the exact value (*i.e.* not a decimal approximation) of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{2^n}$

(10) 3. Suppose that a curve $\mathcal{C}$ given by parametric equations in t passes through the point $(0,2)$ at $t=1$ and satisfies

$$\frac{dx}{dt} = \frac{2}{t} \quad \text{and} \quad \frac{dy}{dt} = 1 - \frac{1}{t^2}$$

(a) Find the parametric equations for $\mathcal{C}$, (*i.e.* for x, y) in terms of t.

(b) Find a cartesian equation for $\mathcal{C}$, (*i.e.* for x, y) by eliminating the parameter in 3(a).

(c) Find the arc length of $\mathcal{C}$ from $t=1$ to $t=2$.

(6) 4. (a) Sketch the graph of $r^2 = \sin\theta$.

(b) Find the area enclosed by this curve.

(5) 5. Sketch and name the following surface, clearly labelling the traces of the graph in the coordinate planes: $x^2 - 4y^2 + z^2 = 1$

(6) 6. Convert the equation $\rho = \cot\phi \csc\phi$ into cylindrical coordinates and into cartesian coordinates. Sketch and name the surface.

(12) 7. A curve is defined by $\boldsymbol{r}(t) = \langle \cos 2t\,,\, \sin 2t\,,\, -2t\rangle$.

(a) Draw a sketch of the curve for $0 \le t \le 2\pi$. *(Hint: this curve lies on a cylinder — draw that cylinder and show the curve on it.)*

(b) Find the velocity and acceleration vectors $\boldsymbol{v}(t), \boldsymbol{a}(t)$, the unit tangent and unit normal vectors $\boldsymbol{T}, \boldsymbol{N}$, and the curvature κ.

(c) Find the length of the curve for $0 \le t \le 2\pi$.

(9) 8. Given $f(x,y,z) = x^2 + y^2 + xyz$:

(a) find the equation of the tangent plane to the surface $f(x,y,z) = 1$ at the point $P(1,1,-1)$;

(b) find the directional derivative of f at P in the direction of $\boldsymbol{v} = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$;

(c) find the maximum rate of change in f at P.

(4) 9. If $z = f(x,y)$ is implicitly defined by $x^2 \ln(yz) + y\,e^x = z \sin(xy)$, find $\dfrac{\partial z}{\partial y}$.

(Marks)

(5) 10. If $f(s)$, $g(t)$ are sufficiently differentiable functions, and $z = f(x+2y) + g(x-2y)$ show that

$$4\,\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2} \ .$$

(6) 11. Find and classify the critical points of $f(x, y) = x^3 - 6xy + y^2$.

(6) 12. Use Lagrange Multipliers to find the shortest distance from the point $(1, 2, 2)$ to the sphere $x^2 + y^2 + z^2 = 36$.

(10) 13. Evaluate the following:

(a) $\displaystyle\iint_R y\, e^{x^3}\, dA$, where R is the triangular region bounded by $y = 2x$, $y = 0$, and $x = 1$.

(b) $\displaystyle\int_0^1 \int_y^1 \cos(x^2)\, dx\, dy$.

(6) 14. (a) Sketch the solid $\mathcal{S}$ which is in the first octant, bounded by the coordinate planes, by the cylindrical surface $x = 4 - y^2$, and by the plane $z = y$.

(b) Find the volume of $\mathcal{S}$.