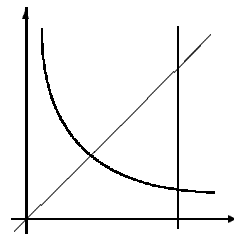


(Marks)

- (8) 1. Let R be the region (as shown) bounded by $y = \frac{2}{x} + 1$, $y = 3x$, and $x = 2$.

- (a) Find the area of R .
- (b) Find the exact volume of the solid that results from revolving R about the y -axis.
- (c) Set up the integral required to find the volume of the solid that results from revolving R about the x -axis.
Do not actually evaluate the integral.



- (4) 2. If $y = x \arctan\left(\frac{1}{x}\right)$ find y' . (Do not simplify.)

- (30) 3. Evaluate the following integrals:

(a) $\int_1^e 2x \ln \sqrt{x} \, dx$ (b) $\int \frac{2 + \sin \theta - \cos \theta \sin \theta}{\cos^2 \theta} \, d\theta$ (c) $\int \frac{x^3}{\sqrt{16 + x^2}} \, dx$

(d) $\int_0^{\pi/3} \sin^3 3x \cos^2 3x \, dx$ (e) $\int \frac{2x^2 - 10x + 7}{(2x + 1)(x - 2)^2} \, dx$ (f) $\int 2x\sqrt{x + 3} \, dx$

- (9) 4. Calculate the following limits

(a) $\lim_{x \rightarrow 0} \frac{xe^x - x}{1 - \cos 2x}$ (b) $\lim_{x \rightarrow 0^+} (1 - 2x)^{1/x^2}$

(c) $\lim_{x \rightarrow +\infty} \frac{\ln(x^4 + 1)}{\ln(x + 1)}$

- (8) 5. Determine whether these improper integrals converge or diverge: if an integral converges, give the exact value of the integral.

(a) $\int_0^1 \frac{x \, dx}{\sqrt{1 - x^2}}$ (b) $\int_1^{\infty} \frac{dx}{(1 + 2x)^3}$

- (5) 6. Find the solution of the following differential equation.

$$xy \frac{dy}{dx} = x + 1, \quad x > 0, \quad y(1) = -2$$

- (3) 7. For the sequence $\{a_k\} = \left\{ \frac{\cos(k\pi)}{e^k} \right\}$, determine whether or not it is convergent. (Justify your answer.)

- (3) 8. Calculate (if possible) the sum of the series $\sum_{n=1}^{\infty} \frac{2}{n^2 + 2n}$

- (12) 9. Classify each of the following series as convergent or divergent. (Briefly justify your conclusions.)

(a) $\sum_{n=0}^{\infty} \frac{n! 2^n}{(2n)!}$ (b) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$

(c) $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n}\right)^n$ (d) $\sum_{n=0}^{\infty} \frac{\sqrt{2n+1}}{n^2+1}$

(Marks)

- (8) 10. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Briefly justify your conclusions.)

(a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$

(b) $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{e^n}$

- (5) 11. Determine the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x+2)^n}{4^{n+1} \sqrt[3]{n+1}}$.

- (5) 12. For the function $f(x) = \frac{1}{x+1}$

- find the first five terms of the Maclaurin series for $f(x)$;
- find the n^{th} term, and express the series in Σ notation.
- What is the radius of convergence for this series?