John Abbott College

# Department of Mathematics

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Final Examination

Mathematics 201 – NYB

## Calculus II Science

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9 points 1. Consider the region bounded by

$$y = \arctan x, \qquad x = 0, \qquad y = \pi/4$$

Figure 1

Set up, but **DO NOT EVALUATE** the integrals to determine:

(a) the area of the bounded region,

(b) and the volume of revolution obtained by rotating the bounded region about

i. the $x$-axis

ii. the $y$-axis

30 points 2. Evaluate each integral.

(a) $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

(b) $\int \arctan x \; dx$

(c) $\int \sin^4 3x \; dx$

(d) $\int \tan^4 x \sec^4 x \; dx$

(e) $\int \dfrac{dx}{\sqrt{4x^2+9}}$

(f) $\int \dfrac{dx}{x(x+1)^2}$

3. Find the following limits:

4 points (a) $\lim\limits_{x \to 1^-} x^{\frac{2}{1-x}}$

3 points (b) $\lim\limits_{x \to \frac{\pi}{2}^-} \dfrac{\sec x}{\tan x}$

5 points 4. Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{\sqrt{x^2+9}}{y^2}\frac{dy}{dx} = x, \qquad y(0) = -1$$

8 points 5. Evaluate the following improper integrals.

(a) Consider the graph of

$$f(x) = \frac{1}{(x-3)^{2/3}}$$

Figure 2

Is it possible to assign a finite number to the area between $f(x)$ and the $x$-axis to the right of $x = 3$ and bounded by $x = 4$? If it is possible find the number.

(b) $\int_{-\infty}^{0} \dfrac{dx}{x^2+1}$

4 points 6. Determine whether the **sequence** is convergent or divergent. If it is convergent, determine its limit.

(a) $a_n = \dfrac{2+n^3}{3+2n^3}$

(b) $a_n = \cos n$

4 points 7. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{4}{(2n-1)(2n+1)}$

16 points 8. Determine whether the following series converge or diverge. State the test used and show that the conditions for the application of the test have been met.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{3n+2}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^{3/2}+1}$

8 points 9. Which does each of the following alternating series do:
converge absolutely, converge conditionally, or diverge? Why?

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n(3^{n+1})}{4^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{3\sqrt{n}+5}$

5 points 10. Find the radius of convergence and the interval of convergence of the following power series.

$$\sum_{n=2}^{\infty} \frac{3^n x^n}{n^2}$$

4 points 11. Find the first four nonzero terms of the Taylor series for $f(x) = \ln x$ centered at $x = 1$.