CEGEP JOHN ABBOTT COLLEGE

Calculus I (Science)

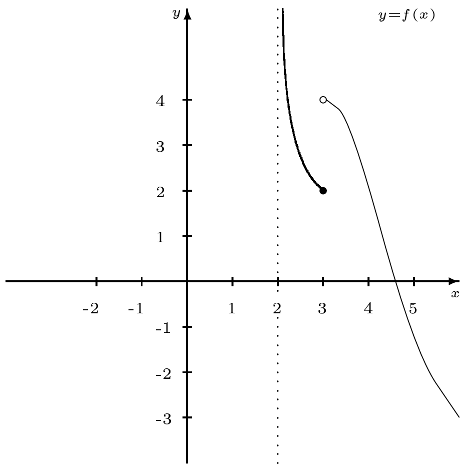
(Mathematics 201–NYA/??)

1. Use algebraic techniques to evaluate the following limits. If a limit fails to exist, assign one of the symbols $+\infty$ or $-\infty$ if possible.

   (a) $\displaystyle\lim_{x\to 5}\frac{2x^2-9x-5}{x^2-3x-10}$ (b) $\displaystyle\lim_{x\to-\infty}\frac{5x^3-6x^5+4}{10x^5+4x+17}$

   (c) $\displaystyle\lim_{x\to\pi}\cos(x-\sin x)$ (d) $\displaystyle\lim_{x\to 5^+}\frac{10}{5-x}$ (e) $\displaystyle\lim_{x\to 1}\frac{\sqrt{x^2+3x}-2}{x-1}$

2. Given the accompanying graph of $y=f(x)$, determine the five items (a)–(e) below.



   (a) $\displaystyle\lim_{x\to 2^+}f(x)$

   (b) $\displaystyle\lim_{x\to 3^+}f(x)$

   (c) $f(3)$

   (d) $\displaystyle\lim_{x\to 3}f(x)$

   (e) $\displaystyle\lim_{x\to 4}f(x)$

3. $f(x)=\dfrac{x}{\tan x}$, use a calculator to

   (a) Fill in the table below:

x (radians)	-0.100	-0.010	-0.001	0.001	0.010	0.100
$f(x)$						

   and

   (b) Estimate $\displaystyle\lim_{x\to 0}\frac{x}{\tan x}$.

4. (a) State the conditions that *define* the continuity of the function f at the number a.

   (b) If $f(x)=\begin{cases}9x^2+4 & \text{if } x\leqslant 0\\ 5-\sin x & \text{if } x>0,\end{cases}$ use the definition in 4(a) to determine whether f is continuous at the number zero.

5. (a) State *the limit definition* of the derivative of the function f.

   (b) If $f(x)=\dfrac{1}{x-3}$, use the definition in 5(a) to calculate $f'(x)$.

6. Differentiate each of the following functions with respect to x. *Do not simplify* your answers.

   (a) $g(x)=6e^x-\dfrac{9}{2x^3}+xe^6+\csc x$ (b) $f(x)=e^{3x}\cos 4x$

   (c) $v(x)=\dfrac{x^2+5x}{x^2-10x+26}$ (d) $y=\left(\sqrt{x}\right)^{\cos x}$

   (e) $u(x)=\sin^3\left(e^{x^5}\right)$

7. If $y=\dfrac{(x^2-5)^3\sqrt{1-x}}{\sqrt[5]{(x+3)^3}}$, use logarithmic differentiation to obtain $\dfrac{dy}{dx}$ as a function of x alone.

8. Calculate the *second* derivative of the function $f(x)=\tan^2 3x$.

9. (a) Sketch the graph of any function, $f(x)$, whose instantaneous rate of change with respect to x is constant for all x.

   (b) Write an equation which defines a function, $g(x)$, whose instantaneous rate of change with respect to x is 3 for all x.

   (c) Write an equation which defines a *nonzero* function, $h(x)$, whose instantaneous rate of change with respect to x is $h(x)$ for all x.

10. Give an *equation* of the straight line tangent to the graph of $y=\dfrac{1}{x^3}$ at the point where $x=-2$. Kindly show supporting work.

11. If $x\sqrt{y}-x^3y^2=14$, use implicit differentiation to obtain $\dfrac{dy}{dx}$.

12. A car is traveling north towards an intersection at a speed of 60 kilometers per hour, while another car is traveling east *away from* the intersection at a speed of 50 kilometers per hour. At what rate is the distance between the two cars changing at the instant when one car is 1 kilometer south of the intersection, and the other is 2 kilometers east of the intersection? [Be sure to say whether the distance between the cars is increasing or decreasing at that instant.]

13. *Without graphing*, determine all values of x for which the graph of the function $f(x)=x\sqrt{50-x^2}$ has horizontal tangents. Show supporting work.

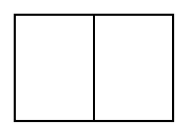
14. *Without graphing*, find the absolute maximum and the absolute minimum attained by the function $f(x)=2+x^{2/3}$ on the interval $[-1,8]$.

15. *Given* that

$$f(x)=\frac{x^2}{(x+2)^2}\,,\quad f'(x)=\frac{4x}{(x+2)^3}\,,\quad f''(x)=\frac{8-8x}{(x+2)^4}\,.$$

   graph $y=f(x)$, showing all relevant steps. Identify any *intercepts, asymptotes*, local *extrema* or *inflection points* the graph may possess. Specify any intervals throughout which the graph is *increasing, decreasing, concave upward* or *concave downward.*

16. A farmer wants to fence an area of 1.5 million square meters in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. If the fencing costs ten dollars per meter, determine the dimensions of the rectangle so as to *minimize* the *cost* of the fence.



17. Perform the indicated integration.

   (a) $\displaystyle\int\frac{x^2-x+1}{\sqrt{x}}\,dx$ (b) $\displaystyle\int_1^2(8x^3+3x^2)\,dx$

   (c) $\displaystyle\int\frac{\cos x}{1-\cos^2 x}\,dx$ (d) $\displaystyle\int\left(3e^u+\sec^2 u+\frac{4}{u}\right)du$

18. Suppose a particle is moving along a straight line with acceleration function $a(t)$, velocity function $v(t)$ and position function $s(t)$. If $a(t)=5\cos t-7\sin t$, $v(0)=4$ *and* $s(0)=3$ construct the position function, $s(t)$.

19. (a) Sketch the region whose area is represented by $\displaystyle\int_3^5(6-x)\,dx$.

   (b) What is the area of the region?