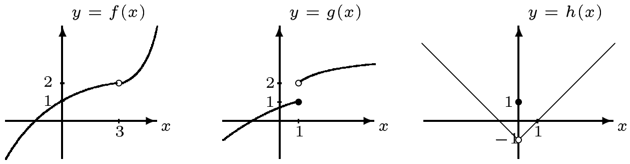
CEGEP JOHN ABBOTT COLLEGE

Calculus I (Science)

(Mathematics 201–NYA/??)

1. (a) Evaluate the limits visually from the respective graphs. If the limit does not exist state in which way ($+\infty$, $-\infty$, or "does not exist").



   (i) $\lim\limits_{x\to 3} f(x)$ (ii) $\lim\limits_{x\to 1} g(x)$ (iii) $\lim\limits_{x\to 0} h(x)$

   (b) Using the above functions, also find,

   (i) $\lim\limits_{x\to 0} f(x)\cdot h(x)$ (ii) $\lim\limits_{x\to 1} g(x)\cdot h(x)$

2. Evaluate the following limits. If the limit does not exist, state in which way ($+\infty$, $-\infty$, or "does not exist").

   (a) $\lim\limits_{x\to 8^+} \sqrt{x-8}$ (b) $\lim\limits_{x\to 3} \dfrac{x-2}{x^2-9}$

   (c) $\lim\limits_{x\to 2} \dfrac{2x^2-7x+6}{x-2}$ (d) $\lim\limits_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3}$

   (e) $\lim\limits_{x\to \frac{\pi}{2}^-} \tan x$ (f) $\lim\limits_{x\to\infty} \dfrac{3x^2-9x+2}{1-x^2}$ (g) $\lim\limits_{x\to\infty} e^{-x}$

3. Evaluate $\lim\limits_{x\to 0} \dfrac{\sin 3x}{x}$. (If you use a calculator, make sure it is in radian mode.)

4. (a) State the conditions under which a function $f(x)$ is continuous at $x = c$.

   (b) Is $g(x)$ continuous at $x = 1$? Justify your answer.

$$g(x) = \begin{cases} -2x+3 & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$$

5. (a) State the limit definition of the derivative.

   (b) Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{3}{1-x}$.

6. Find $\dfrac{dy}{dx}$ for each of the following functions. Do not simplify.

   (a) $y = \pi^x + \dfrac{x}{\pi} + x^\pi + \dfrac{\pi}{x} + \pi$ (b) $y = e^{2\sin x} + \sin^2(e^x)$

   (c) $y = \tan^3\left(\dfrac{x+2}{x-2}\right)$ (d) $y = \left(x^4+6x\right)^{\cos x}$

   (e) $y = 3^{\sin x} - \log_4(7x-1) + \ln^3 x$

   (f) $y = \dfrac{x^4}{3} - \dfrac{5}{\sqrt[3]{x^2}} + \cos\left(\dfrac{\pi}{7}\right)$ (g) $y = \sqrt{x^3+1}\,(\sec x)$

7. Find $\dfrac{dy}{dx}$ if $\sin(xy) + 4 = x^2 + y^2$.

8. Find the equation of the tangent line to the curve $y = x^3 - 3x^2$ at $x = 1$.

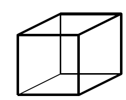
9. Given $y = 5x^3 + e^{x^2}$, find y''.

10. Sketch the graph that satisfies these conditions:
    - $f(-3) = 4$, $f(-1) = -1$, $f(2) = 1$, $f'(-1) = 0$
    - $f'(x) = 0$ when $x < -3$
    - $f'(x) < 0$ when $-3 < x < -1$
    - $f'(x) > 0$ when $x > -1$
    - $f''(x) > 0$ when $-3 < x < 2$;
    - $f''(x) < 0$ when $x > 2$.

11. Sketch the following graph showing all steps. *Label* all intercepts, relative extrema, inflection points and horizontal and vertical asymptotes. State the intervals on which the function is increasing, decreasing, concave up, and concave down.

$$f(x) = \frac{1+x^2}{1-x^2}, \quad f'(x) = \frac{4x}{\left(1-x^2\right)^2}, \quad f''(x) = \frac{4\left(1+3x^2\right)}{\left(1-x^2\right)^3}.$$

12. Canadian postal regulations require that the sum of the three dimensions of a rectangular packgage does not exceed 3m. For a package with square ends, what are the dimensions of the package of maximum volume that can be mailed?



13. A ball is thrown straight upward from the earth's surface with an intial velocity of 100 ft/sec. The position of the ball as a function of time (measured in seconds) is given by $s(t) = -16t^2 + 100t$.

    (a) Determine the average velocity of the ball between 2 and 3 seconds.

    (b) Determine the velocity function.

    (c) What was the instantaneous velocity of the ball exactly 2.5 seconds after it was thrown?

14. Find the absolute extrema (maximum and minimum) of the function $f(x) = x^3 + 2x^2 - 4x + 1$ on the closed interval $[-3, 0]$.

15. Evaluate the following.

    (a) $\displaystyle\int \frac{(x^2-1)(x^2+3)}{x^3}\,dx$ (b) $\displaystyle\int \left(e^x + \tan^2 x\right)dx$

    (c) $\displaystyle\int_0^{\frac{\pi}{2}} (\sin x - \cos x)\,dx$ (d) $\displaystyle\int_1^e \left(e^x - \frac{1}{x}\right)dx$

16. Given $\dfrac{dy}{dx} = 4x^2 - \dfrac{15}{x^2}$, find y if $y = 45$ when $x = 3$.

17. Find the area of the region bounded by the graph of $y = x^3 - 4x + 5$, the x-axis, and the lines $x = 0$ and $x = 5$.