Examinations
from previous years

CEGEP
JOHN ABBOTT
COLLEGE

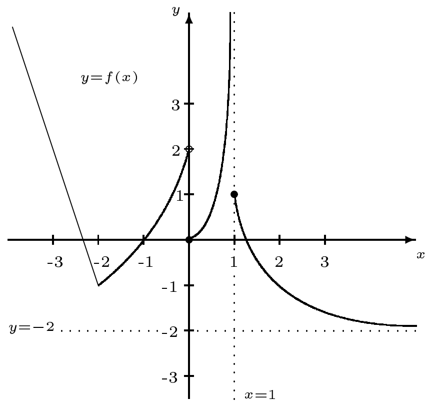
Autumn 1998

Calculus I (Science)

(Mathematics 201–NYA/??)

1. Estimate the limit numerically: $\lim\limits_{x\to 0} \dfrac{1-e^{x^2}}{x^2}$.

2. Refer to the sketch below to evaluate the following limits. If a value does not exist, state in which way ($+\infty$, $-\infty$, or "does not exist").



   (a) $\lim\limits_{x\to -\infty} f(x)$
   (b) $\lim\limits_{x\to -2} f(x)$
   (c) $\lim\limits_{x\to 0} f(x)$
   (d) $\lim\limits_{x\to 1^-} f(x)$
   (e) $\lim\limits_{x\to 1^+} f(x)$
   (f) $\lim\limits_{x\to \infty} f(x)$

3. Calculate the following limits (if they exist). Make your answer as informative as possible: if a limit does not exist, say so; if appropriate one-sided limits exist instead, state them explicitly; if any limits are infinite, state this explicitly as well.

   (a) $\lim\limits_{x\to\infty} \dfrac{2x+7}{5x-x^2}$ (b) $\lim\limits_{x\to -3^-} \dfrac{|x+3|}{x+3}$ (c) $\lim\limits_{x\to 0} \dfrac{\sqrt{2x+4}-2}{x}$

   (d) $\lim\limits_{x\to 2} \dfrac{2x^2-5x+2}{x^2-4}$ (e) $\lim\limits_{x\to 1^-} \dfrac{x^2+1}{1-x}$

4. Find the value of c for which $f(x) = \begin{cases} cx+1 & \text{if } x \leqslant 3 \\ cx^2-1 & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.

5. Sketch, if possible, the graph of a function that is continuous but not differentiable at $x = 2$. If this is not possible, explain why.

6. State a limit definition of the derivative. Use this definition to find the derivative of $f(x) = x^2 - 3x$.

7. For each of the following functions, calculate the derivative $\dfrac{dy}{dx}$. Do not simplify your answers.

   (a) $y = 7x^3\sqrt{x} + \dfrac{5}{x} + \sqrt[6]{x^5} - 4\pi$ (b) $y = \dfrac{e^{x^3-x}}{2+7x^2}$

   (c) $y = 2\cos x - \sqrt{9-\sin^2 x}$ (d) $y = e^{\sec x}\tan(2x)$

   (e) $y = \left(x+e^{2x}\right)^{5x}$

8. For $f(x) = x\sqrt{98-x^2}$:
   (a) find $f'(x)$ and simplify your answer;
   (b) find the values of x for which the tangent line is horizontal.

9. Find an equation for the line tangent to the graph of $y = \dfrac{8}{\sqrt{4+3x}}$ at the point $(4, 2)$.

10. Given $xe^{x-y^2} = x^2 - y^2$, find the slope of the tangent line to the curve at $(2, \sqrt{2})$.

11. Given $y = \ln\left(1+x^2\right)$, find and simplify the second derivative y'' or $\dfrac{d^2y}{dx^2}$.

12. The position (in metres) of a particle at time t (seconds) is given by the equation $x = \dfrac{t^2}{200} + \ln(t+1)$. Find the velocity when the acceleration is 0. (Assume $t \geqslant 0$.)

13. Evaluate the following integrals:

    (a) $\displaystyle\int \frac{x^5+2x-\sqrt[3]{x}}{x^4}\,dx$ (b) $\displaystyle\int \left(\frac{2}{t} + e^t - \cos t\right) dt$

    (c) $\displaystyle\int \left(2+x^2\right)^2 dx$ (d) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\varphi + \frac{2}{\sin^2\varphi}\right) d\varphi$

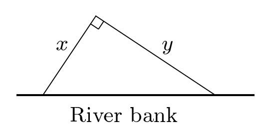
14. Given that $f'(x) = 2 - 4x$, and $f(1) = 5$, find $f(x)$.

15. Find the area of the region which lies between the curve $y = 16 - x^4$ and the x-axis.

16. An object is moving with an acceleration given by the equation $a(t) = 16 - t^2$. (t is time in seconds, $t \geqslant 0$, a is acceleration in m/sec^2.) At what time is the velocity of the object maximal?

17. Determine whether or not the function $S(x) = \dfrac{1}{x} + x^2$ has a maximum value. If it does, what is the maximum value? If you think it does not have a maximum, justify your claim.

18. An enclosure in the form of a right-angled triangle is constructed using some fencing along two-sides, and the river bank along the hypotenuse, as shown in the diagram. If 400 m of fencing is available, what are the dimensions x and y that maximize the area of the enclosure?



19. Find the vertical and horizontal asymptotes to the graph of $f(x) = \left(\dfrac{2x-77}{4+300x}\right)^2$.

20. For the function $f(x) = x\sqrt{1-x^2}$, the first and second derivatives are

$$f'(x) = \frac{1-2x^2}{\sqrt{1-x^2}} \quad \text{and} \quad f''(x) = \frac{x(2x^2-3)}{(1-x^2)^{3/2}} :$$

    (a) find the intervals where the function is increasing, and the intervals where it is decreasing;
    (b) find the intervals where the function is concave up, and the intervals where it is concave down;
    (c) find the coordinates of all relative (or local) extreme points;
    (d) find the coordinates of all points of inflection;
    (e) sketch the graph.

    Make sure that your graph clearly illustrates all these features.