Examinations
from previous years

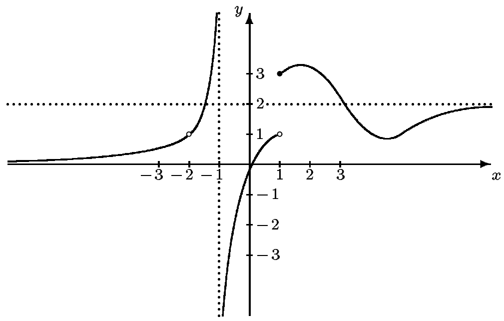
CEGEP
JOHN ABBOTT
COLLEGE

Calculus I (Science)

(Mathematics 201–NYA/??)

Autumn 1994

1. Consider the function f whose graph is given below:



For each of the following limits, assign a finite number, or ∞, or $-\infty$, or state that the limit does not exist.

(a) $\lim\limits_{x\to-\infty} f(x)$ (b) $\lim\limits_{x\to-2} f(x)$ (c) $\lim\limits_{x\to-1^+} f(x)$
(d) $\lim\limits_{x\to-1} f(x)$ (e) $\lim\limits_{x\to1^-} f(x)$ (f) $\lim\limits_{x\to1^+} f(x)$
(g) $\lim\limits_{x\to1} f(x)$ (h) $\lim\limits_{x\to\infty} f(x)$

2. Determine the following limits. All questions are to be done using analytic reasoning and/or algebraic techniques. (Show your work.)

(a) $\lim\limits_{x\to-2} \dfrac{x^2+3x+2}{x^3-x^2-6x}$ (b) $\lim\limits_{x\to\infty} \dfrac{x^2+2x-5}{2x^2-6x-1}$

(c) $\lim\limits_{t\to0^-} \sqrt{t\cos t}$ (d) $\lim\limits_{y\to-2^+} \dfrac{2}{y^2-4}$

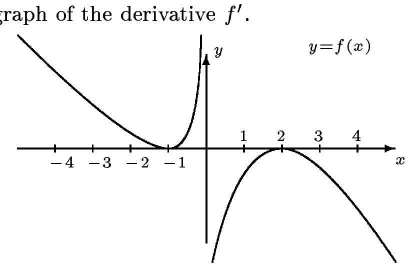
3. (a) State a (limit) definition for the derivative $f'(x)$ of a given function $f(x)$.
(b) Use your definition from part (a) to find $f'(x)$ for the function $f(x) = 6x - x^2$.

4. (a) State the definition for a function f to be continuous at the number a.
(b) Use your definition from part (a) to determine if

$$f(x) = \begin{cases} 2^{3x}-2 & \text{for } x<1 \\ \ln x + 2 & \text{for } x \geqslant 1 \end{cases}$$

is continuous at $a = 1$.
(c) Explain why $f(x) = \dfrac{x^2+6x-16}{x+8}$ is *not continuous* at $a = -8$.

5. Consider the function f whose graph is given below. Use this to sketch the graph of the derivative f'.



6. Find the (first) derivative for each of the following functions. *Do not simplify* your answers.

(a) $y = 3x^5 + \dfrac{2}{3x} - \sqrt[5]{x^3} + 6\pi^2$ (b) $f(t) = \sin\left(\dfrac{3s-2}{t^2+1}\right)$

(c) $g(x) = 2^x + \log_2(4x)$ (d) $u = \sqrt{w}\cos^2 w$

(e) $f(x) = (\ln x)^{x^2}$

7. Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answers.
(a) Use logarithmic differentiation: $y = \dfrac{x^3(3x-1)^4}{(2x+1)\sqrt{x^2+4}}$.
(b) Use implicit differentiation: $x^2y + \tan y = x^2 + y^2$.

8. Find an equation for the line tangent to the graph of $y = x + e^{x^2}$ at the point where $x = 1$.

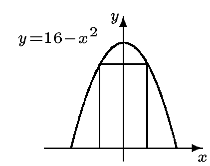
9. Consider $f(x) = x(3x+1)^{1/3}$.
(a) Find $f'(x)$ and simplify your answer.
(b) Use part (a) to find all values of x, if any, for which the tangent line to the graph of f is horizontal.

10. Consider $f(x) = \ln(x^2+1)$.
(a) Find $f''(x)$ and simplify your answer.
(b) Use part (a) to find all values of x, if any, at which there is a change in concavity (*i.e.*, an inflection point).

11. Gravel is being dumped from a conveyor belt at a rate of 30 m^3/hr, and it forms a pile in the shape of a cone whose height is always twice the radius of the base. How fast is the height of the pile increasing when the pile is 6m high? (The volume of a cone with height h and radius r is $\frac{1}{3}\pi r^2 h$.)

12. Find the (absolute) maximum and minimum values of the function $f(x) = x + \dfrac{9}{x}$ on $[1, 8]$.

13. Find the dimensions of the rectangle of largest area with its base on the x-axis and its other two corners above the x-axis and on the parabola $y = 16 - x^2$.



14. Given $f(x) = x^{1/3}(4+x)$,

$$f'(x) = \frac{4x+4}{3x^{2/3}} \quad \text{and} \quad f''(x) = \frac{4x-8}{9x^{5/3}}.$$

(a) Find and specify all intervals where f is increasing; decreasing; concave up; and concave down.
(b) Determine the coordinates of any relative extreme values and any points of inflection.
(c) Sketch a graph of f, showing all information obtained in parts (a) and (b).

15. Consider a function f with the following properties:

$$\lim_{x\to-\infty} f(x) = 1, \quad \lim_{x\to\infty} f(x) = \infty, \quad \text{and} \quad f(0) = 1$$

and with the characteristics given in the table below:

	$-\infty<x<-1$	$x=-1$	$-1<x<2$	$x=2$	$2<x<\infty$
$f(x)$		undefined		$f(2)=3$	
$f'(x)$	positive	does not exist	positive	$f'(2)=0$	positive
$f''(x)$	positive	does not exist	negative	$f''(2)=0$	positive

Sketch the graph of a function f which satisfies all of the above characteristics.

16. Find the following indefinite integrals:

(a) $\displaystyle\int (2x^3 - 3e^x + 3^2)\,dx$ (b) $\displaystyle\int \frac{1+\sin\vartheta}{\cos^2\vartheta}\,d\vartheta$ (c) $\displaystyle\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)dx$

17. Evaluate the following definite integrals; express your answers without using decimals.

(a) $\displaystyle\int_0^{\frac{\pi}{3}} (4\cos t - 3\sin t)\,dt$ (b) $\displaystyle\int_{-e}^{-1} \frac{2+x}{x^2}\,dx$

18. Find the area enclosed by the graph $y = 4x - x^2$, $y = 0$, $y = 1$, $x = 1$, and $x = 4$.