MATH 009 **FINAL EXAM** **WINTER 2002**

1. Simplify each expression and give answers with positive exponents.

   (a) $\dfrac{(3^{-1}x^2y^{-3})^{-3}}{9x^{-4}y^{-5}}$ (3)

   (b) $x^{\frac{3}{2}}\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$ (1)

2. Perform the following division:

   (a) $\dfrac{x^2 - 4x^{\frac{1}{2}}}{\sqrt{x}}$ (1)

   (b) $\dfrac{25x^3 + 10x^2 - 8x + 13}{5x - 1}$ (3)

3. Rationalize the numerator and simplify your answer.

   $\dfrac{\sqrt{7} - \sqrt{5}}{2}$ (2)

4. Factor completely:

   (a) $5x^3 - 20x$ (2)

   (b) $4ax + 2x^2y - 6ay - 3xy^2$ (2)

   (c) $8a^3 - 27$ (2)

5. Give exact values for the following radical expressions:

   (a) $\dfrac{\sqrt[3]{(-8)^2}}{\sqrt[4]{16}}$ (1)

   (b) $8 + 3\sqrt{27} - \sqrt{48} - \left(2\sqrt{2}\right)^2$ (1)

6. Perform the following operations and simplify your answers:

   (a) $(2x - 3)^2 + 8x - 4x^2 - 2(x + 1)$ (2)

   (b) $\dfrac{x^2 - x - 6}{2x^2 + 5x - 3} \div \dfrac{2x^2 + 5x + 2}{4x^2 - 1}$ (4)

   (c) $\dfrac{2}{(x - 1)^2} + \dfrac{1}{x - 1} - \dfrac{1}{x}$ (4)

7. Given A$(-3, 2)$ and B$(5, 7)$, find:

   (a) distance d between A and B (1)

   (b) midpoint M of the line segment $\overline{AB}$ (1)

   (c) slope m of the line through A and B (1)

   (d) equation of the line through A and B. Give $y = mx + b$ form. (2)

(e) equation of a line that is perpendicular to the line AB and passing through the point B. Give $ax + by + c = 0$ form. (2)

8. Solve the following:

(a) $3x^2 - 2x - 3 = 0$ (2)

(b) $\sqrt{3x+1} - 1 = x$ (2)

(c) $\dfrac{x}{x-1} - \dfrac{2}{x} = \dfrac{1}{x^2 - x}$ (3)

(d) $\dfrac{2}{3}x - 5 \geq \dfrac{1}{3}$ (2)

(e) $\log(x+7) - \log(x-2) = 1$ (2)

(f) $8^{x+2} = 16$ (2)

9. Given: $f(x) = x^2 - 7x + 3$

(a) find and simplify $\dfrac{f(x+h) - f(x)}{h}$ (3)

(b) evaluate your answer in part (a) when $h = 0$. (1)

10. If $f(x) = \sqrt{x}$ and $g(x) = 2x + 1$ find:

(a) $f(g(x))$ (1)

(b) $f(g(4))$ (1)

(c) $g^{-1}(x)$ (1)

11. Sketch the graph and state the domain and range:

(a) $y = -x^2 + 6x - 5$ (4)

(b) $y = \begin{cases} x^2 - 1 & x < 1 \\ -2 & x \geq 1 \end{cases}$ (3)

12. Sketch the graph and give the coordinates of all intercepts and the equations of all asymptotes:

(a) $y = \dfrac{2-x}{x+1}$ (4)

(b) $y = \log_2(x-2)$ (3)

13. Give the exact value for $3^{\log_3 5} + \log_3 81 - \ln e^3$ (1)

14. Write $\ln\left(\dfrac{x^8 y^9}{\sqrt{z}}\right)$ using simple logarithms. (1)

15. Given $5^x = 3$

    (a) Express the equation in log form (1)

    (b) Find x correct to 3 decimal places. (1)

16. ΔABC has $\angle C = 90°$, side $c = 7$ and side $b = 5$.
    Find the exact value for $\sin A$ and $\sec A$ (2)

17. Approximate to 4 decimal place accuracy:

    (a) $\cot 134°$ (1)

    (b) $\sin 2.9$ (1)

    (c) the acute angle θ with $\sec\theta = 2.3$ (Answer in degrees) (1)

18. Give the exact value for the following. Use reference angles where applicable.

    (a) $\cos 225°$ (1)

    (b) $\tan\left(-\frac{7\pi}{6}\right)$ (1)

    (c) the angle $\frac{5}{12}$ radians, converted to degrees. (Answer in terms of π) (1)

    (d) $\sin\theta$, given that $\tan\theta = -\frac{5}{4}$ and θ is in Quadrant IV. (1)

    (e) θ in Quadrant II, given that $\sin\theta = \frac{\sqrt{3}}{2}$ (1)

19. Verify the following indentities:

    (a) $\sec^2\theta + \tan^2\theta = 1 + 2\tan^2\theta$ (2)

    (b) $\frac{\sec\theta - 1}{1 - \cos\theta} = \sec\theta$ (2)

20. Given: $y = -2\cos(3x)$.

    (a) find the Amplitude and Period. (1)

    (b) sketch 2 cycles of the graph (3)

21. ΔABC has $\angle C = 90°$, $\angle A = 70°$and side $c = 13$. Solve the ΔABC.
    Give all answers correct to 2 decimal places. (3)

22. ΔABC has $\angle B = 108°$, $\angle A = 29°$and side $c = 20$. Find side a. (3)

23. ΔABC has side $a = 12$, side $b = 10$ and side $c = 15$. Find $\angle A$. (3)

Answers:

1 $(a)\ \dfrac{3y^{14}}{x^2}$ $(b)\ x^2 - x$

2 $(a)\ x^{3/2} - 4$ $(b)\ 5x^2 + 3x - 1 + \dfrac{12}{5x-1}$

3 $\dfrac{1}{\sqrt{2}+\sqrt{5}}$

4 $(a)\ 5x\,(x+2)\,(x-2)$ $(b)\ (2x-3y)\,(2a+xy)$ $(c)\ 2a-3\,(4a^2+6a+9)$

5 $(a)\ 2$ $(b)\ 5\sqrt{3}$

6 $(a)\ -6x+7$ $(b)\ \dfrac{x-3}{x+3}$ $(c)\ \dfrac{3x-1}{x\,(x-1)^2}$

7 $(a)\ \sqrt{89}$ $(b)\ \left(1, \dfrac{9}{2}\right)$ $(c)\ \dfrac{5}{8}$ $(d)\ y = \dfrac{5}{8}\,x + \dfrac{31}{8}$ $(e)\ 8x+5y-75=0$

8 $(a)\ \dfrac{1 \pm \sqrt{10}}{3}$ $(b)\ 0, 1$ (c) No solution $(d)\ x \geq 8$ $(e)\ 3$ $(f)\ -\dfrac{2}{3}$

9 $(a)\ 2x+h-7$ $(b)\ 2x-7$

10 $(a)\ \sqrt{2x+1}$ $(b)\ 3$ $(c)\ y = \dfrac{1}{2}x - \dfrac{1}{2}$

11(a) FIGURE

11(b) FIGURE

12(a) $X-int = (2,0)\,,\ Y-int = (0,2)\ ,\ VA: x=-1,\ \ HA: y=-1$
FIGURE

12(b) FIGURE

13 $5+4-3=6$

14 $8\ln x + 9\ln y - \dfrac{1}{2}\ln z$

15 $(a)\ x = \log_5 3$ $(b)\ \ x = \dfrac{\ln 3}{\ln 5} = .683$

16 $\sin A = \dfrac{2\sqrt{6}}{7}$ and $\sec A = \dfrac{7}{5}$

17 $(a)\ -.9657$ $(b)\ .2392$ $(c)\ 64.2°$

18 $(a)\ \dfrac{-1}{\sqrt{2}}$ $(b)\ \dfrac{-1}{\sqrt{3}}$ $(c)\ \dfrac{75}{\pi}$ $(d)\ \dfrac{-5}{\sqrt{41}}$ $(e)\ 120°$ for $0° \leq \theta < 360°$

19(a) $LHS = 1 + \tan^2\theta + \tan^2\theta = 1 + 2\tan^2\theta = RHS.$ N.B. $\sec^2\theta = 1 + \tan^2\theta$

19(b)
$$\begin{aligned} LHS &= \frac{\frac{1}{\cos\theta} - 1}{1 - \cos\theta} \\ &= \frac{\left(\frac{1}{\cos\theta} - 1\right)}{(1 - \cos\theta)} \times \frac{\cos\theta}{\cos\theta} \\ &= \frac{1 - \cos\theta}{(1 - \cos\theta)\cos\theta} \\ &= \frac{1}{\cos\theta} \\ &= \sec\theta \\ &= RHS \end{aligned}$$

20(a) (a) $A = 2$ and $P = \frac{2\pi}{3}$

20(b) FIGURE

21 $\angle B = 20°$, $a = 12.22$ and $b = 4.45$

22 $a = 14.21$

23 $\angle A = 52.8°$