

## Algebra and Trigonometry—Mathematics 201–009 Fall 2001 Final Examination

1. Simplify each expression. Express answers with positive exponents.

$$(a) -3^{-2}xz^4(4x^3z^{-2})^3 \quad (b) \left(\frac{4a^{-1}b^{-3}}{16a^4b^{-2}}\right)^{-2} \quad (c) \left(\frac{125}{8}\right)^{-2/3}$$

2. Rationalize the denominator and simplify:
- $\frac{21}{\sqrt{7}-2}$

3. Simplify the radical expression:
- $\sqrt{27} - \sqrt{75} + 2\sqrt{12}$

4. Simplify:
- $(3x - 4)^2 - (2x + 5)^2$

5. Factor each polynomial completely.

$$(a) 27x^3 + 1 \quad (b) 2x^2 - 5x - 3 \quad (c) x^3 - xy^2 + 2ax^2 - 2ay^2$$

6. Simplify.

$$(a) \frac{x^2 - 16}{x^2 + 5x + 4} \div \frac{x^2 - 8x + 16}{x^2 - 1} \quad (b) \frac{2}{x-3} - \frac{5}{(x-3)^2} + \frac{4}{x^2-9}$$

7. (a) Find an equation of the line passing through the points
- $P(2, -1)$
- and
- $Q(5, 3)$
- .

(b) Find an equation of the line passing through  $P(2, -1)$  and parallel to the  $x$ -axis.(c) Find an equation of the line with  $x$ -intercept equal to 4 and  $y$ -intercept equal to  $-2$ .(d) Find an equation of the line passing through  $P(2, -1)$  and perpendicular to the line with equation  $y = -3x + 4$ .

8. Sketch the graph of each of the following lines. Find all intercepts and state the slope for each.

$$(a) 5x - y = 0 \quad (b) y = -2$$

9. Given the points
- $A(2, -3)$
- and
- $B(4, 7)$
- .

(a) Find the midpoint of the line segment  $AB$ .(b) Find the length of the line segment  $AB$ .

10. Solve each equation.

(a)  $2x^2 - 8x + 5 = 0$

(b)  $x^4 - 10x^2 + 9 = 0$

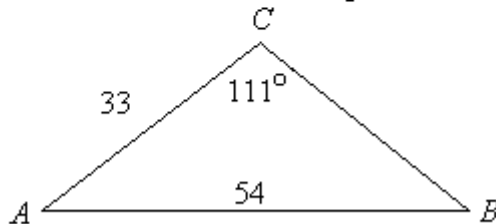
(c)  $\sqrt{5x-6} = 7$

(d)  $\frac{x}{x-2} + \frac{2}{x+2} = 1$

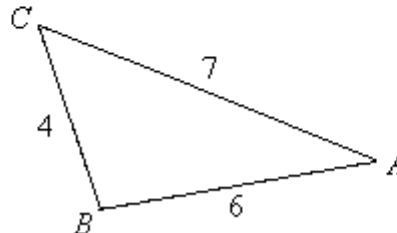
11. Solve the inequality:  $1 - 3x \leq 7$
12. Given  $f(x) = x^2 + 3x - 5$ . Simplify:  $\frac{f(x+h) - f(x)}{h}$
13. Sketch the graph of  $y = -x^2 + x$ . Give the domain, range, coordinates of intercepts and the vertex.
14. Sketch the graph of  $f(x) = \frac{3x - 5}{x + 2}$ . Give the domain, coordinates of intercepts and equations of asymptotes.
15. Sketch the graph of  $y = 5^x - 5$ . Give the domain, range, coordinates of intercepts and equations of asymptotes.
16. Sketch the graph of  $f(x) = 1 + \ln x$ . Give the domain, range, coordinates of intercepts and equations of asymptotes.
17. Simplify.  
(a)  $\log_2 64$  (b)  $e^{\ln x}$
18. Evaluate: accurate to four decimal places.  
(a)  $\log 200$  (b)  $\ln 0.0098$  (c)  $\log_8 888$  (d)  $e^4$
19. Write using a single logarithm:  $\ln 8 + 3 \ln x - 5 \ln y$
20. Write using simple logarithms:  $\log \left[ \frac{16(x+4)}{x^3} \right]$
21. Solve each equation for  $x$ . If decimals are needed, be accurate to four decimal places.  
(a)  $\ln(x+1) + \ln(x-1) = \ln 99$  (b)  $e^{2x-1} = 8888$
22. Verify the identity:  $\frac{\csc^2 x}{\cot x} - \frac{\csc x}{\cos x} = 0$
23. Let  $y = 5 \sin 4x$   
(a) Find the amplitude. (b) Find the period.  
(c) Sketch two cycles of the graph.

24. Scott, David and Talicia go into a cornfield in late summer. They meant to cut down the corn and make an equilateral triangle, but it was dark and cutting corn is not easy. Each of them stands at one corner of the triangle. Scott is 25 m from Talicia and 30 m from David. The angle in the triangle where Scott stands is  $65^\circ$ . How far is Talicia from David? Give your answer accurate to two decimal places.

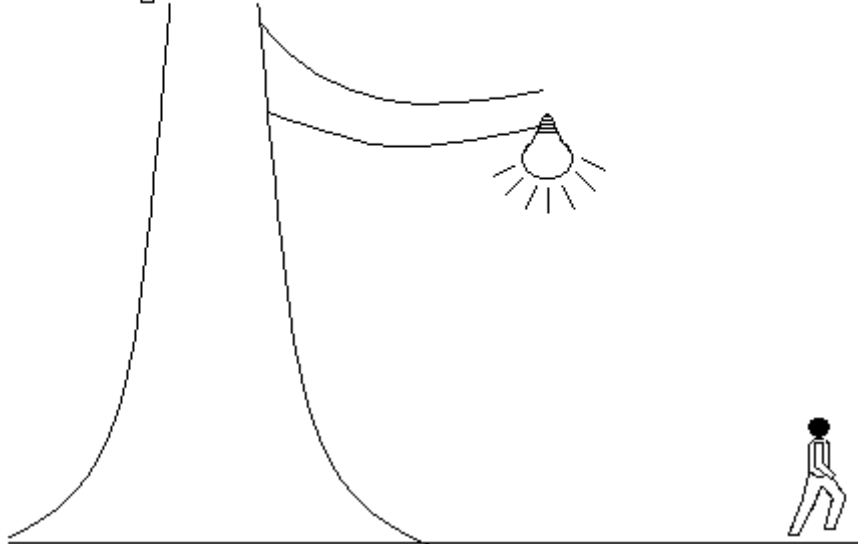
25. Find the value of angle  $B$  accurate to two decimal places:



26. Find angle  $A$  accurate to two decimal places:



27. Matt is 175 cm tall. He is walking along a dark and lonely street in Hudson, and the only light is cast by a dim bulb hanging from the branch of a dead tree. His shadow is 2 m long when he has walked 5 m from directly under the bulb. How high is the bulb from the ground?



## Answers

1. (a)  $-\frac{64x^{10}}{9x^2}$   
 (b)  $16a^{10}b^2$   
 (c)  $\frac{4}{25}$

2.  $7\sqrt{7} + 14$

3.  $2\sqrt{3}$

4.  $5x^2 - 44x - 9$

5. (a)  $(3x + 1)(9x^2 - 3x + 1)$

(b)  $(x + 2a)(x - y)(x + y)$

(c)  $(2x + 1)(x - 3)$

6. (a)  $\frac{x - 1}{x - 4}$

(b)  $\frac{2x^2 - x - 45}{(x - 3)^2(x + 3)}$

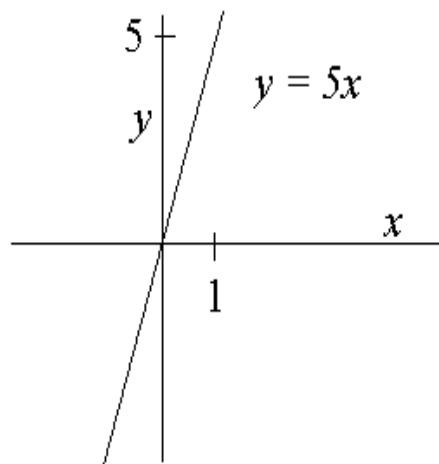
7. (a)  $y = \frac{4}{3}x - \frac{11}{3}$

(b)  $y = -1$

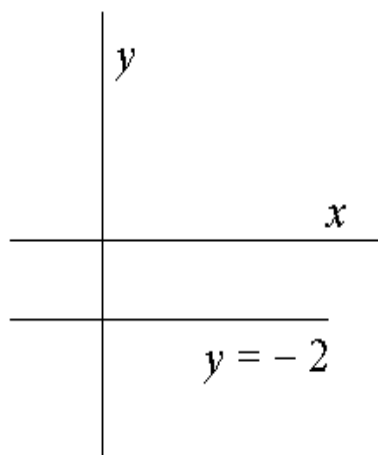
(c)  $y = \frac{1}{2}x - 2$

(d)  $y = \frac{1}{3}x - \frac{5}{3}$

8. (a)  $x\text{-int}=(0, 0)$   
 $y\text{-int}=(0, 0)$   
 $m = 5$



(b)  $y\text{-int}=(0, -2)$   
 $m = 0$



9. (a)  $(3, 2)$

(b)  $2\sqrt{26}$

10. (a)  $\frac{4 \pm \sqrt{6}}{2}$

(b)  $\pm 1, \pm 3$

(c) 11

(d) 0

11.  $x \geq -2$

12.  $2x + h + 3$

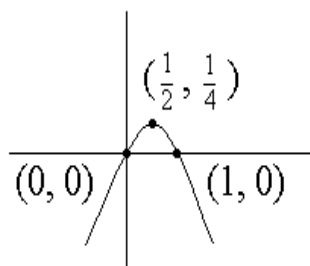
13. Domain =  $\mathbf{R}$

Range =  $(-\infty, \frac{1}{4})$

$x$ -intercepts =  $(0, 0), (1, 0)$

$y$ -intercept =  $(0, 0)$

Vertex =  $(\frac{1}{2}, \frac{1}{4})$



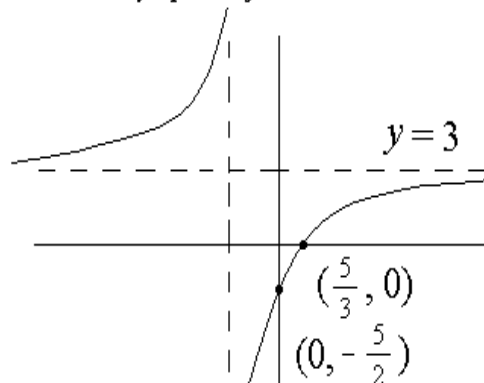
14. Domain =  $\mathbf{R} - \{-2\}$

$x$ -intercepts =  $(\frac{5}{3}, 0)$

$y$ -intercept =  $(0, -\frac{5}{2})$

Vertical asymptote:  $x = -2$

Horizontal asymptote:  $y = 3$



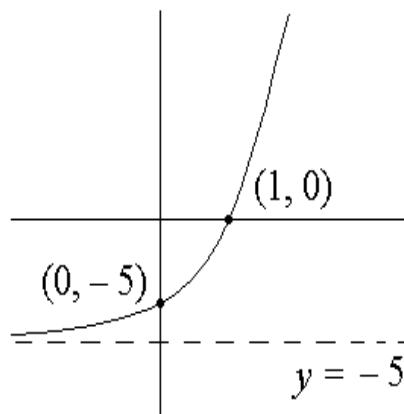
15. Domain =  $\mathbf{R}$

Range =  $(-5, \infty)$   $x = -2$

$x$ -intercepts =  $(1, 0)$

$y$ -intercept =  $(0, -4)$

Horizontal asymptote:  $y = -5$

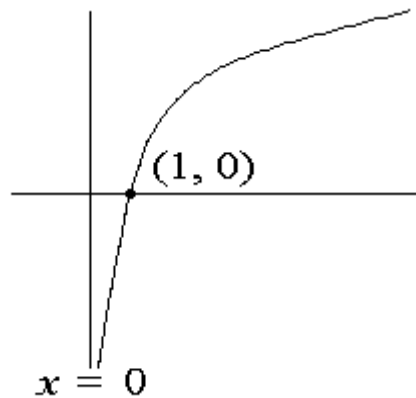


16. Domain =  $(0, \infty)$

Range =  $\mathbf{R}$

$x$ -intercepts =  $(\frac{1}{e}, 0)$

Vertical asymptote:  $x = 0$



17. (a) 6

(b)  $x$

18. (a) 2.3010

(b)  $-4.6254$

(c) 3.7890

(d) 54.5982

19.  $\ln \frac{8x^2}{y^5}$

20.  $\log 16 + \log(x + 4) - 3 \log x$

21. (a) 10

(b) 5.0462

$$22. \frac{\csc^2 x}{\cot x} - \frac{\csc x}{\cos x}$$

$$= \frac{1}{\frac{\sin^2 x}{\cos x}} - \frac{1}{\cos x}$$

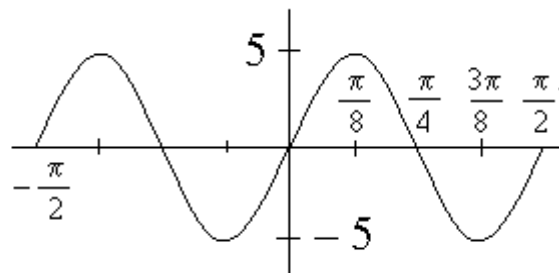
$$= \frac{\cos x}{\sin^2 x} - \frac{1}{\cos x}$$

$$= \frac{\cos^2 x}{\cos x \sin^2 x} - \frac{1}{\cos x \sin^2 x} = 0$$

23. (a) 5

(b)  $\frac{\pi}{2}$

(c)



24. 29.85 m

25. 34.79°

26. 34.77°

27. 6.125 m