CEGEP JOHN ABBOTT COLLEGE

INTRODUCTION TO ALGEBRA

(MATHEMATICS 201-007/∞)

1. Simplify each of the following. (All variables represent positive numbers.)

   (a) $2(30-2(3^2+7^0))-5^2+|-10|$ (b) $\dfrac{(3x^2y^{-2})^3}{(6xy^{-3})^2}$

   (c) $\left(\dfrac{3ab^2}{c^4}\right)^2\left(\dfrac{9ab^3}{c^5}\right)^{-1}$

   (d) $xy^2\sqrt{18x}+\sqrt{8x^3y^4}-y\sqrt{50x^3y^2}$ (e) $\sqrt{6x^3y^3}\sqrt{12xy^2}$

2. Rationalize the denominator and express your answer in lowest terms.

   (a) $\dfrac{4}{3+\sqrt{7}}$ (b) $\dfrac{x}{1-\sqrt{1-x}}$

3. Perform the indicated operations and express your answer in standard form.

   (a) $-7x^3+5x^2+2+3x(2x^2-3x+4)$
   (b) $(3t+2)^2-(2t-3)(2t+3)$
   (c) $(5x^2-8x+9)(x^2+2x+3)$

4. Factor each of the following polynomials.

   (a) $28r^2t-63s^2t$ (b) $12x^3y+12x^2y+3xy$
   (c) $x^2-5x-204$ (d) $3z^3-2z^2-27z+18$

5. Perform the long division: $\dfrac{12x^4+7x^3-37x^2-25x+13}{4x-7}$

6. Perform the indicated operations and express your answer in lowest terms.

   (a) $\dfrac{x^2+x}{x^2-x-2}\cdot\dfrac{3x^2-5x-2}{6x^2+2x}$ (b) $\dfrac{2x^2-x-15}{x^2+4x-21}\div\dfrac{4x^2-25}{3x^2+19x-14}$

   (c) $\dfrac{t}{2t-3}+\dfrac{1}{5t+9}-\dfrac{3(t^2+6)}{10t^2+3t-27}$

7. Solve each of the following equations.

   (a) $2(7x+8)-2x=3(5x+2)+1$ (b) $(5z+4)^2=81$
   (c) $10x^2+11x=6$ (d) $9x^2=6x+5$
   (e) $\dfrac{8-w}{3w^2+7w-6}=\dfrac{w}{3w-2}-\dfrac{1}{w+3}$ (f) $x=5+\sqrt{2x-7}$

8. Find the slope, the x-intercept and the y-intercept of the line whose equation is $4x-5y=12$, and sketch the graph of this line.

9. Given the equations of the lines

   $\ell_1: 14x+4y=23, \quad \ell_2: x=-\dfrac{2}{7}y-8, \quad \ell_3: 2x-7y=9.$

   (a) Which pairs of lines are parallel? Justify.
   (b) Which pairs of lines are perpendicular? Justify.

10. Give an equation of the line which
    (a) passes through the points $(-2,5)$ and $(1,3)$.
    (b) has slope $-\frac{7}{5}$ and passes through the point $(8,-4)$.
    (c) passes through the point $(6,4)$ and is perpendicular to the line $x+3y=9$.
    (d) has slope 2 and y-intercept $(0,-3)$.

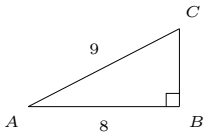
11. Solve each system of linear equations.

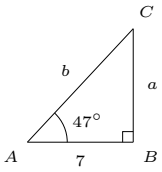
    (a) $\left\{\begin{array}{rcrcr} 7x & + & 14y & = & -21 \\ -5x & - & 10y & = & 10 \end{array}\right.$ (b) $\left\{\begin{array}{rcrcr} 4x & + & 3y & = & 8 \\ 2x & - & y & = & 2 \end{array}\right.$

12. (a) A rectangle is two-thirds as wide as it is long, and its perimeter is 15 inches. What is the area of the rectangle?
    (b) The height of a triangle is two centimetres less than twice the length of its base. The area of the triangle is 12 square centimetres. What is the height of the triangle? How long is the base of the triangle?
    (c) The sum of four consecutive positive whole numbers is equal to the product of the smallest number and the largest number. What are the numbers?

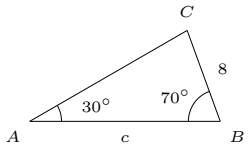
13. A ladder is leaning against the top of a 12 foot wall. If the bottom of the ladder is 5 feet away from the wall, find the angle that the ladder makes with the ground and the length of the ladder. (Give the angle to the nearest hundredth of a degree.)

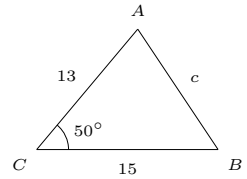
14. For the triangle in part (a), give the exact values of $\sin A$, $\cos A$ and $\tan A$. For the triangle in part (b), find a and b accurate to two decimal places.

    (a)
    

    (b)
    

15. Solve for c in each of the following triangles. Round your answer to nearest hundredth.

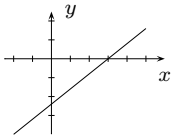
    (a)
    

    (b)
    

16. A surveyor marks points A and B 200 metres apart on one bank of a river. She sights a point C on the opposite bank and determines that angle CAB is 80° and angle CBA is 36°. Find the distance from A to C. Round your answer to the nearest tenth of a metre.

ANSWERS

1. (a) 5, (b) $\frac{3}{4}x^4$, (c) ab/c^3, (d) 0, (e) $6x^2y^2\sqrt{2y}$.
2. (a) $2(3-\sqrt{7})$, (b) $1+\sqrt{1-x}$.
3. (a) $-x^3-4x^2+12x+2$,
   (b) $5t^2+12t+13$,
   (c) $5x^4+2x^3+8x^2-6x+27$.
4. (a) $7t(2r-3s)(2r+3s)$,
   (b) $3xy(2x+1)^2$, (c) $(x-17)(x+12)$,
   (d) $(z-3)(z+3)(3z-2)$.
5. $3x^3+7x^2+3x-1+\dfrac{6}{4x-7}$
6. (a) $\frac{1}{2}$, (b) $\dfrac{3x+2}{2x-5}$, (c) $\dfrac{t+7}{5t+9}$.
7. (a) $x=3$, (b) $z=-\frac{13}{5}$, 1,
   (c) $x=-\frac{3}{2}$, $\frac{2}{5}$; (d) $x=\frac{1}{3}(1\pm\sqrt{6})$,
   (e) $w=2$, (f) $x=8$.
8. x-intercept: $(3,0)$
   y-intercept: $\left(0,-\frac{12}{5}\right)$
   slope: $\frac{4}{5}$

9. $m_1=m_2=-\frac{7}{2}$, $m_3=\frac{2}{7}$.
   $\therefore$ (a) $\ell_1\parallel\ell_2$, (b) $\ell_1\perp\ell_3$, $\ell_2\perp\ell_3$.
10. (a) $2x+3y=11$, (b) $7x+5y=36$,
    (c) $3x-y=14$, (d) $2x-y=3$.
11. (a) The system has no solution (*i.e.*, it is inconsistent). (b) $\left(\frac{7}{5},\frac{4}{5}\right)$
12. (a) The area of the rectangle is $13\frac{1}{2}$ square inches. (b) The triangle is 6 cm high, and its base is 4 cm long. (c) The numbers are 3, 4, 5 and 6.
13. The ladder is 13 feet long, and it makes an angle of approximately 67.38° with the ground.
14. (a) $\sin A=\frac{1}{9}\sqrt{17}$, $\cos A=\frac{8}{9}$,
    $\tan A=\frac{1}{8}\sqrt{17}$. (b) $a=7\tan 47^\circ\approx 7.51$,
    $b=7/\cos 47^\circ\approx 10.26$.
15. (a) $c=8\sin 80^\circ/\sin 30^\circ\approx 15.76$,
    (b) $c=\sqrt{394-390\cos 50^\circ}\approx 11.97$.
16. The distance from A to C is
    $200\sin 36^\circ/\sin 64^\circ\approx 130.8$ metres.