CEGEP JOHN ABBOTT COLLEGE

INTRODUCTION TO ALGEBRA

(MATHEMATICS 201-007/∞)

1. Find the exact value of each of the following *Show your work and do not give decimal answers*.
   (a) $\left(1\frac{1}{3}\right)\left(5\frac{1}{2}\right) \div \frac{7}{6}$ (b) $\dfrac{-3^0(-2)^5+1}{3|4-20|}$

2. Simplify. *Give answers with positive exponents.*
   (a) $\dfrac{(x^8y^{-4})^2}{x^{-2}y^5}$ (b) $\left(\dfrac{4xy^2}{x^{-1}y}\right)^{-2}$

3. Perform the indicated operations and simplify.
   (a) $-3(2t+4)+5(2t-4)$ (b) $(5x-1)(2x+y)$
   (c) $(m^2+5)(4m^3-2m^2+4m)$ (d) $\dfrac{2x^3+x^2-2x-2}{x+1}$

4. Factor completely.
   (a) $2xy-3y-8x+12$ (b) $3m^2+8m-3$
   (c) $4y^5+12y^4-40y^3$ (d) $16k^4-1$

5. Perform the indicated operations and write each answer in lowest terms.
   (a) $\dfrac{x^2-4}{(x+3)(x-2)} \cdot \dfrac{-2x}{(x+2)(x+3)}$ (b) $\dfrac{m^2-4}{m^2-1} \div \dfrac{2m^2+4m}{1-m}$
   (c) $\dfrac{4}{r^2-r}+\dfrac{6}{r^2+2r}$ (d) $\dfrac{5}{x^2-9}-\dfrac{x+2}{x^2+4x+3}$

6. Solve the following equations.
   (a) $4(x+8)=2(2x+6)+20$ (b) $\frac{1}{2}(x+2)+\frac{3}{4}(x+4)=x+5$
   (c) $x^2-5x=-6$ (d) $5x^2-x-1=0$
   (e) $10y^2=-5y$ (f) $(3k-1)^2=16$
   (g) $\dfrac{5-2y}{y}=\dfrac{1}{4}$ (h) $\dfrac{x}{4-x}=\dfrac{2}{x}$

7. Using algebraic techniques and defining variables, solve the following problems.
   (a) If three is added to a number and that sum is doubled, the result is two more than the number. Find the number.
   (b) If three U.S. dollars can be exchanged for 4.5204 Canadian dollars, how many Canadian dollars can we get for $49.20 U.S.? What is the exchange rate?
   (c) A ladder leans against an upright wall so that the distance from its bottom to the wall is 2 feet less than the distance from its top to the ground. Give an algebraic expression for the length of the ladder.

8. For the line with equation $3x-2y=8$,
   (a) find the x-intercept, the y-intercept and the slope, and
   (b) graph the line.

9. Give an equation for the line
   (a) with slope $-\frac{2}{3}$ and passing through the point $(-4,1)$.
   (b) passing through the points $(-1,-2)$ and $(-3,-8)$.

10. Given the equations for $\begin{cases} \ell_1\colon\ x-3y=1, \\ \ell_2\colon\ y=-3x+4 \text{ and} \\ \ell_3\colon\ 3x-5=9y. \end{cases}$
    (a) Which lines are parallel? Justify.
    (b) Which lines are perpendicular? Justify.

11. Solve the system and state your conclusion. $\begin{cases} x-2y=8 \\ 3x+4y=6 \end{cases}$

12. Simplify each of the following. *Do not give decimal answers.*
    (a) $\sqrt{24}+6\sqrt{54}$ (b) $(\sqrt{10}-\sqrt{7})(2\sqrt{10}+3\sqrt{7})$
    (c) $\sqrt{48x^2y}+5x\sqrt{27y}$ (Assume $x,\ y \geqslant 0$.) (d) $\sqrt{5}\sqrt{15}-4\sqrt{3}$

13. Rationalize the denominator and simplify your answer: $\dfrac{6}{5-\sqrt{2}}$

14. Use a calculator to estimate the following with 4 decimal place accuracy.
    (a) $\cos 35^\circ$ (b) $\sec 69^\circ$ (c) $\cot\dfrac{5\pi}{4}$ (d) $\csc\dfrac{\pi}{3}$

For questions 15–17, express your answer with three decimal place accuracy.

15. $\triangle ABC$ has $\angle C=90^\circ$ with side $c=9$ and $\angle A=39^\circ$. Find side a.

16. $\triangle ABC$ has $\angle A=74^\circ$, $\angle B=34^\circ$ and side $c=11$. Find side a.

17. $\triangle ABC$ has $\angle A=110^\circ$, side $b=3$ and side $c=6$. Find side a.

ANSWERS

1. (a) $\frac{44}{7}$, (b) $\frac{11}{16}$.

2. (a) $\frac{x^{18}}{y^{13}}$, (b) $\frac{1}{16x^4y^2}$.

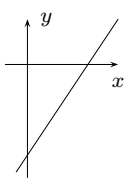
3. (a) $4t-32$, (b) $10x^2+5xy-2x-y$,
   (c) $4m^5-2m^2+24m^3-10m^2+20m$,
   (d) $2x^2-x-1+\frac{1}{x+1}$.

4. (a) $(y-4)(2x-3)$, (b) $(3m-1)(m+3)$,
   (c) $4y^3(y+5)(y-2)$,
   (d) $(2k-1)(2k+1)(4k^2+1)$.

5. (a) $\frac{-2x}{(x+3)^2}$, (b) $\frac{2-m}{2m(m+1)}$, (c) $\frac{2(5r+1)}{r(r-1)(r+2)}$,
   (d) $\frac{11+6x-x^2}{(x-3)(x+1)(x+3)}$.

6. (a) $\mathbb{R}$, (b) $x=4$, (c) $x=2,\ 3$,
   (d) $x=(1\pm\sqrt{21})/10$, (e) $y=0,\ -\frac{1}{2}$,
   (f) $k=-1,\ \frac{5}{3}$, (g) $\frac{20}{9}$, (h) $x=-4,\ 2$.

7. (a) The number is 4. (b) About $74.13 Canadian (one Canadian dollar is worth approximately 66.4 percent of a U.S. dollar). (c) The length of the ladder is $\sqrt{2x^2+4x+4}$ in terms of the distance, x, from its bottom to the wall, or $\sqrt{2y^2-4y+4}$ in terms of the distance, y, from its top to the ground.

8. 

   $3x-2y=8$
   x-intercept: $(\frac{8}{3},0)$
   y-intercept: $(0,-4)$
   Slope: $\frac{3}{2}$

9. (a) $2x+3y+5=0$, (b) $3x-y-1=0$.

10. ℓ_1 and ℓ_3 are parallel to each other, and perpendicular to ℓ_2.

11. $x=22/5, y=-9/5$.

12. (a) $20\sqrt{6}$, (b) $\sqrt{70}-1$, (c) $19x\sqrt{3y}$, (d) $\sqrt{3}$.

13. $6(5+\sqrt{2})/23$.

14. (a) 0.8192, (b) 2.7904, (c) 1, (d) 1.1547.

15. $a=9\sin 39^\circ \approx 5.664$.

16. $a=(11\sin 74^\circ)/\sin 72^\circ \approx 11.118$.

17. $a=3\sqrt{5-4\cos 110^\circ} \approx 6.934$.