INTRODUCTION TO POLARIZED CATEGORIES

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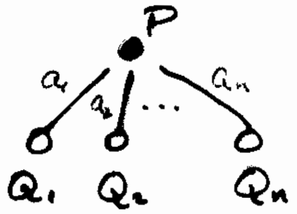
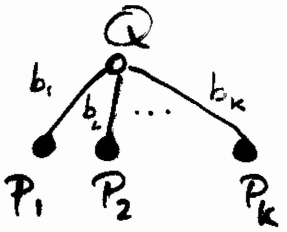
Basic finitary AJ Games



$$P = \bigsqcup_{i \in I} Q_i = \{ a_i : Q_i \mid i \in I \} = \left\{ \begin{array}{c} a_1 : Q_1 \\ \vdots \\ a_n : Q_n \end{array} \right\}$$

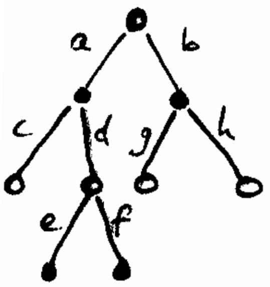


$$Q = \prod_{j \in J} P_j = (b_j : P_j \mid j \in J) = \left(\begin{array}{c} b_1 : P_1 \\ \vdots \\ b_k : P_k \end{array} \right)$$

Examples

$\bullet \qquad 0 = \bigsqcup \emptyset = \{ \}$

$\circ \qquad 1 = \prod \emptyset = ( )$



$$\{ a : (c : \{\}, d : \{ e : (), f : () \}), \; b : (g : \{\}, h : \{\}) \}$$

Maps

O-maps: $(b_j \mapsto h_j)_{j \in J} : Q \to \prod_J P_j = (b_j : P_j)_J$

TUPLE

for $h_j : Q \to P_j$, an OP map

P-maps: $\{a_i \mapsto h_i\}_{i \in I} : \{a_i : Q_i\}_I = \bigsqcup_I Q_i \longrightarrow P$

COTUPLE

for $h_i : Q_i \longrightarrow P$, an OP map

OP-maps: $\overrightarrow{a_K} \cdot g : Q \longrightarrow \{a_i : Q_i\}_I$ for $g : Q \to Q_K$

INJECTION

PROJECTION

$\overleftarrow{b_K} \cdot f : (b_j : P_j)_J \longrightarrow P$ for $f : P_K \to P$

Examples

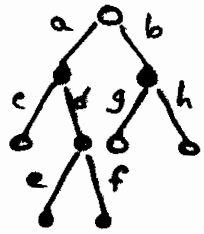
1) 

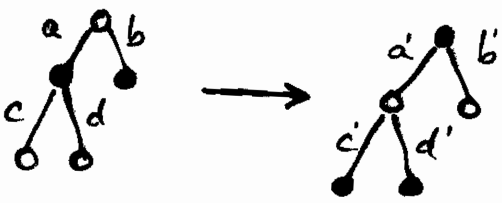
ie $\begin{pmatrix} p : \{\} \\ q : \{\} \end{pmatrix} \longrightarrow \{\}$

Two projections:

$\overleftarrow{p} \cdot 1$ and $\overleftarrow{q} \cdot 1$

$1 : \{\} \to \{\}$ is the "null" $\{\}$ identity map

2) $\begin{pmatrix} a \mapsto \vec{c} \cdot () \\ b \mapsto \vec{g} \cdot () \end{pmatrix} : \circ \longrightarrow$ 

3) 

<u>Exercise</u>: Find all maps between these games.

Here's one:

$\vec{a'} \cdot \begin{pmatrix} c' \mapsto \overleftarrow{b} \cdot \{\} \\ d' \mapsto \overleftarrow{b} \cdot \{\} \end{pmatrix}$

Compositions & Rewrites

$$g \,;\, (b_j \mapsto h_j) \implies (b_j \mapsto g\,;h_j)$$

$$Q \xrightarrow{h_j} P_j$$

$$\begin{tikzcd} & Q \arrow[dr, "(h_j)"] & \\ Q' \arrow[ur, "g"] \arrow[rr, "(g;h_j)"'] & & \prod_j P_j \end{tikzcd}$$

$$Q' \xrightarrow[g]{} Q \xrightarrow[h_j]{} P_j$$

$$(b_j \mapsto h_j);\overleftarrow{b_k}\cdot f \implies h_k\,;f$$

$$Q \xrightarrow{h_j} P_j \qquad P_k \xrightarrow{f} P$$

$$\begin{tikzcd} Q \arrow[r, "(h_j)"'] \arrow[rr, bend right, "h_k;f"] & \prod_j P_j \arrow[r, "\overleftarrow{b_k}\cdot f"] & P \end{tikzcd}$$

$$Q \xrightarrow[h_k]{} P_k \xrightarrow[f]{} P$$

$$\overleftarrow{b_k}\cdot f \,;\, f' \implies \overleftarrow{b_k}\cdot(f;f')$$

$$P_k \xrightarrow{f} P$$

$$\begin{tikzcd} & P \arrow[dr, "f'"] & \\ \prod_j P_j \arrow[ur, "\overleftarrow{b_k}\cdot f"] \arrow[rr, "\overleftarrow{b_k}\cdot(f;f')"'] & & P' \end{tikzcd}$$

$$P_k \xrightarrow[f]{} P \xrightarrow[f']{} P'$$

And duals for $\sqcup$

Missing (eg):

Given $P_K \xrightarrow[h_i]{} R_i \quad (i \in I)$

form $\prod_j P_j \longrightarrow R_i \quad (i \in I)$

+ so $\prod_j P_j \longrightarrow \prod_i R_i$

And $P_K \longrightarrow \prod R_i$

+ so $\prod_j P_j \longrightarrow \prod_i R_i$

$$(\overleftarrow{b_K}\cdot h_i)_i \underset{?}{=} \overleftarrow{b_K}\cdot(h_i)_i$$

- The rewrite sstm is confluent + terminates
- The associative law (for maps) is satisfied
- There are canonical 'identity' maps
  (= units for composition)

$$\mathrm{id}_{\{a_i : Q_i\}} := \{\vec{a}_i \cdot 1_{Q_i}\}$$

(induction)

- So we have: categories $\underline{X}_O$, $\underline{X}_P$

O-games + maps

P-games + maps

and a module $\hat{X}$

OP-maps

an $\underline{X}_O$-$\underline{X}_P$ module

This is our basic logical & categorical structure

The Basic Logic

3 kinds of sequents

$P \vdash_P P'$ player sequents

$Q \vdash_O Q'$ opponent sequents

$Q \underset{O\,P}{\longmapsto} P$ Cross (mixed, OP) sequents

Sequent Rules

$$\frac{}{A \vdash_P A} \qquad \frac{}{A \vdash_O A} \qquad \text{(atomic identities)}$$

$$\frac{\{\, Q_i \underset{O\,P}{\longmapsto} P \,\}_{i \in I}}{\bigsqcup Q_i \vdash_P P} \qquad \frac{\{\, Q \underset{O\,P}{\longmapsto} P_j \,\}_{j \in J}}{Q \vdash_O \prod P_j} \qquad \begin{pmatrix}\text{cotuple} \\ + \\ \text{tuple}\end{pmatrix}$$

$$\frac{Q \vdash_O Q_K}{Q \underset{O\,P}{\longmapsto} \bigsqcup Q_i} \qquad \frac{P_K \vdash_P P}{\prod P_j \underset{O\,P}{\longmapsto} P} \qquad \begin{pmatrix}\text{injection} \\ + \\ \text{projection}\end{pmatrix}$$

$$\frac{P_0 \vdash_P P_1 \quad P_1 \vdash_P P_2}{P_0 \vdash_P P_2} \qquad \frac{Q_0 \vdash_O Q_1 \quad Q_1 \vdash_O Q_2}{Q_0 \vdash_O Q_2}$$

$$\frac{Q \underset{O\,P}{\longmapsto} P \quad P \vdash_P P'}{Q \underset{O\,P}{\longmapsto} P'} \qquad \frac{Q' \vdash_O Q \quad Q \underset{O\,P}{\longmapsto} P}{Q' \underset{O\,P}{\longmapsto} P} \qquad \text{(cuts)}$$

Cut elimination
Church Rosser

via a term calculus (like $\Sigma\Pi$)

The Categorical Doctrine

A polarized category ($\underline{X}_o$, $\underline{X}_p$, $\hat{\underline{X}}$) consists of categories $\underline{X}_o$, $\underline{X}_p$ and a module $\hat{\underline{X}}$ between them.

$\hat{\underline{X}}$ is a profunctor $\underline{X}_o \longrightarrow \underline{X}_p$; equiv a set of formal arrows closed under "composition" – think 'actions'

Example: $\mathcal{G}(\underline{C}, \mathcal{K})$ where $\underline{C}$ is a category
I, J distinguished objects of $\underline{C}$
$\mathcal{K}$ a subset of $\underline{C}(I,J)$

$\mathcal{G}_o$: Obj : pairs $(\mathcal{R}, X)$ where $\mathcal{R} \subseteq \underline{C}(I,X)$, X an obj of $\underline{C}$

Mor : f : $(\mathcal{R}, X) \xrightarrow{f} (\mathcal{R}', X')$ where $X \xrightarrow{f} X'$ in $\underline{C}$ satisfying

$r \in \mathcal{R} \Rightarrow r;f \in \mathcal{R}'$

$$\begin{tikzcd} I \arrow[d, "\mathcal{R} \ni r"'] \arrow[dr, "\in \mathcal{R}'"] & \\ X \arrow[r, "f"'] & X' \end{tikzcd}$$

$\mathcal{G}_p$:

$$\begin{tikzcd} (Y, \mathcal{S}) \arrow[d, "g"] \\ (Y', \mathcal{S}') \end{tikzcd} \qquad \begin{tikzcd} Y \arrow[d, "g"'] \arrow[dr, "\in \mathcal{S}"] & \\ Y' \arrow[r, "s' \in \mathcal{S}'"'] & J \end{tikzcd}$$

$\hat{\mathcal{G}}$:

$$\begin{tikzcd} (\mathcal{R}, X) \arrow[d, "h"] \\ (Y, \mathcal{S}) \end{tikzcd} \qquad \begin{tikzcd} X \arrow[d, "h"'] & I \arrow[l, "r"'] \arrow[d, dashed, "\in \mathcal{K}"] \\ Y \arrow[r, "s"'] & J \end{tikzcd}$$

$\forall r \in \mathcal{R}, s \in \mathcal{S} : r;h;s \in \mathcal{K}$

"orthogonality"
$f \perp g \leftrightarrow f;g \in \mathcal{K}$

A polarized functor $F = \langle F_o, F_p, \hat{F} \rangle : (\underline{X}_o, \underline{X}_p, \hat{\underline{X}}) \to (\underline{X}_o', \underline{X}_p', \hat{\underline{X}}')$ consists of functors $F_o : \underline{X}_o \to \underline{X}_o'$, $F_p : \underline{X}_p \to \underline{X}_p'$ and a module morphism $\hat{F} : \hat{\underline{X}} \to \hat{\underline{X}}'$

ie $\quad X \underset{m}{\not\to} Y \mapsto F_o(X) \underset{\hat{F}(m)}{\not\to} F_p(Y)$ (closed under 'actions')

A polarized nat. transformation $\alpha = \langle \alpha_o, \alpha_p \rangle : F \to F'$ consists of $\alpha_o : F_o \to F_o'$, $\alpha_p : F_p \to F_p'$ (nat. trans)

sat:

$$\begin{tikzcd}
F_o(A) \arrow[r, "\hat{F}(m)"] \arrow[d, "\alpha_o(A)"'] & F_p(B) \arrow[d, "\alpha_p(B)"] \\
F_o'(A) \arrow[r, "\hat{F}'(m)"'] & F_p'(B)
\end{tikzcd}$$

for any $m : A \not\to B$ in $\hat{\underline{X}}$

Thus a 2-cat : $\underline{\text{PolCat}}$

- • o O • • -

What's missing? The 'game operations' $\sqcup$ and $\sqcap$

<u>Note</u> these are **NOT** the notion of Σ and Π in the 2-cat $\underline{\text{PolCat}}$

(Think of the typing, eg)

Inner & Outer Adjoints

Start with what we want for $\sqcup$ and $\sqcap$:

$\underline{X}$ has I-indexed polarized products (I a (finite) set) if there is a functor

$$\sqcap_I : \underline{X}_p^I \longrightarrow \underline{X}_o \qquad \text{so that}$$

$$\frac{\{X \xrightarrow{f_i} Y_i\}_{i \in I} \quad \text{in } \hat{\underline{X}}}{X \xrightarrow[(f_i)_I]{} \sqcap_I Y_i \quad \text{in } \underline{X}_o}$$

Dually for I-indexed polarized sums

- Chosen products ...
- universal property does exist
- arbitrary polarized lim + colim

So polarized sums & products amount to having:

a pair of functors $\sqcap_I$, $\sqcup_I$ (in a sense "adjoint" to 'diagonal')

DEF Given $F : \underline{X} \to \underline{Y}$, a polarized functor

$G_o : \underline{Y}_p \to \underline{X}_o$ and $G_p : \underline{Y}_o \longrightarrow \underline{X}_p$, (ord.) functors

G not polarized

F has an inner adjoint G

G has an outer adjoint F

if

nat. bij

$$\frac{F_o(X) \longrightarrow Y' \quad \text{in } \hat{\underline{Y}}}{X \longrightarrow G_o(Y') \quad \text{in } \underline{X}_o} \qquad \frac{Y \longrightarrow F_p(X') \quad \text{in } \hat{\underline{Y}}}{G_p(Y) \longrightarrow X' \quad \text{in } \underline{X}_p}$$

Think: F is 'diagonal': $\underline{X} \to \underline{X}^I$

G_o is $\sqcap_I$

G_p is $\sqcup_I$

Inner/Outer adjoints are specified by a universal property:

A pol. functr $F : \underline{X} \to \underline{Y}$ has an inner adjoint iff

there are object functions $G_o : Y_p \to X_o$, $G_p : Y_o \to X_p$

& natural families of module arrows

$$\epsilon_{Y'} : F_o G_o(Y') \nrightarrow Y'$$

$$\eta_Y : Y \longrightarrow F_p G_p(Y)$$

so $\forall\, g : F_o(X) \nrightarrow Y'$ and $f : Y \nrightarrow F_p(X')$ $\exists!\ g^b, f^\#$ as:

$$\begin{tikzcd}
F_o(X) \arrow[d, "F_o(g^b)"'] \arrow[dr, "g", "|" marking] & \\
F_o G_o(Y') \arrow[r, "\epsilon_{Y'}"', "|" marking] & Y'
\end{tikzcd}
\qquad
\begin{tikzcd}
Y \arrow[r, "\eta_Y", "|" marking] \arrow[dr, "f"', "|" marking] & F_p G_p(Y) \arrow[d, "F_p(f^\#)"] \\
& F_p(X')
\end{tikzcd}$$

So inner adjoints are unique up to a unique iso

- $\underline{X}$ has I-indexed polarized sums & products iff $\Delta_I : \underline{X} \longrightarrow \underline{X}^I$ has an inner adjoint

[So $\sqcup$ and $\sqcap$ are unique ---]

- The module $\hat{X}$ (in a pol. cat.) is given by an adjunction $(\ )^* \dashv (\ )_*$ iff $1_{\underline{X}} : \underline{X} \longrightarrow \underline{X}$ has an inner adjoint

$$\begin{array}{c}
Q \to P_* \ (\text{in } \underline{X}_o) \\ \hline\hline
Q^* \to P \ (\text{in } \underline{X}_p) \\ \hline\hline
Q \nrightarrow P \ (\text{in } \hat{X})
\end{array}$$

given by singleton $\sqcup$, $\sqcap$:

$Q^* = \sqcup_1 Q$, $P_* = \sqcap_1 P$

"add a top node"

A boat-load of additives

Suppose $\hat{X}$ is given by $()^* \dashv ()_*$

"<u>AFT-additives</u>" Suppose $\underline{X}_o$ has products $\wedge$ [of obj of form P_*]

and $\underline{X}_P$ has sums $\vee$ [of object of form Q^*]

Then we can construct polarized sums + products:

$$\sqcap_J P_j = \bigwedge_J P_{j*}$$

$$\sqcup_I Q_i = \bigvee_I Q_i^*$$

"additive AFTer polarity"

"<u>FORE-additives</u>" Suppose we have sums Σ in $\underline{X}_o$
and products Π in $\underline{X}_P$

can add these freely "Fam construction"

Then we can construct polarized sums + products:

$$\sqcap_J P_j = (\Pi P_j)_*$$

$$\sqcup_I Q_i = (\Sigma Q_i)^*$$

"additive beFORE polarity" ...

... Laurent-style additives

Summary of Typing

FORE	$\Pi P = P$	$\Sigma Q = Q$
POL	$\sqcap P = Q$	$\sqcup Q = P$
AFT	$\wedge Q = Q$	$\vee P = P$

Example $\mathcal{G}(\underline{C}, -\mathcal{K}) := \mathcal{G}$

- $\hat{\mathcal{G}}$ given by $(\ )^* \dashv (\ )_*$:

$$(\mathcal{R}, X)^* = (X, \mathcal{R}^*), \quad \mathcal{R}^* = \{ h : X \to I \mid r \perp h \ \forall r \in \mathcal{R} \}$$

$$(Y, \mathcal{S})_* = (\mathcal{S}_*, Y), \quad \mathcal{S}_* = \{ k : I \to Y \mid k \perp s \ \forall s \in \mathcal{S} \}$$

Exercise

$$\frac{\dfrac{(\mathcal{R}, X)^* \longrightarrow (Y, \mathcal{S})}{(\mathcal{R}, X) \longrightarrow\!\!\!\!\!+ \ (Y, \mathcal{S})}}{(\mathcal{R}, X) \longrightarrow (Y, \mathcal{S})_*}$$

- $\mathcal{G}$ has a full boat-load of additives (if $\underline{C}$ has sums & products)

→ $\bigsqcup_I (\mathcal{R}_i, X_i) = \left(\Sigma_i X_i \, , \{ h : \Sigma X_i \to I \mid r_i ; b_i \perp h \ \ \forall r_i \in \mathcal{R}_i \} \right)$

→ $\sum_I (\mathcal{R}_i, X_i) = \left(\bigcup_I (\mathcal{R}_i ; b_i) \, , \Sigma_i X_i \right)$

→ $\bigwedge_I (\mathcal{R}_i, X_i) = \left(\langle \mathcal{R}_1 \cdots \mathcal{R}_n \rangle \, , \Pi_i X_i \right)$

(& duals)

"Lots of Egs" :

$$\begin{tikzcd} \underline{\mathrm{PolCat}} \arrow[r, bend right, "\mathrm{Gam}"'] \arrow[r, phantom, "\top"] & \underline{\mathrm{PolGam}} \arrow[l, bend right, "u"'] \end{tikzcd}$$

Robin

Beyond the Basic Logic

1^{st} add "context" [polycategorical structure]

P-seq $\quad \Gamma / P \setminus \Gamma' \vdash_P \Delta$

O-seq $\quad \Gamma \vdash_O \Delta / Q \setminus \Delta'$

Mixed seq $\quad \Gamma \underset{O \quad P}{\longmapsto} \Delta$

All well-typed cut rules (there are 24 (!))

Polarized Polycategories -(the "obvious" categorical version of this)

(i.e. add cut-elimination rewrites)

Circuits (aka "proof nets")

The additives fit nicely into this context (!)

Eg polarized $\sqcup$ and $\sqcap$:

$$\prod_{i \in I} \hat{\underline{X}}(\Gamma, X_i, \Gamma'; \Delta) \cong \underline{X}_p(\Gamma / \textstyle\bigsqcup_i X_i \backslash \Gamma'; \Delta)$$

$$\prod_{j \in J} \hat{\underline{X}}(\Gamma; \Delta, Y_j, \Delta') \cong \underline{X}_o(\Gamma; \Delta / \textstyle\bigsqcap_j Y_j \backslash \Delta')$$

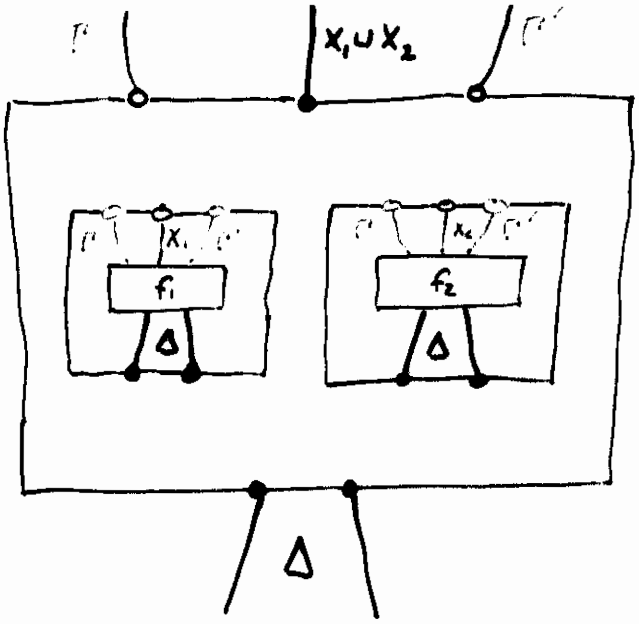
<u>PolPolyCat</u> : 2-cat of polarized polycats, ("evident" 1- + 2-cells ...)

<u>PolPolyGam</u> : 2-cat of pol. <u>game</u> polycats (ie with pol. additives) (& 1+2 cells)

$$\begin{tikzcd} \mathrm{PolPolyCat} \arrow[r, bend right, "\mathrm{Gam}"'] \arrow[r, phantom, "\top"] & \mathrm{PolPolyGam} \arrow[l, bend right, "U"'] \end{tikzcd}$$

$\mathrm{Gam}(\underline{X})$ = "formal games on $\underline{X}$-objects" (defined inductively)

[Binary ⊔ via circuits:]



Fore & Aft additives have a similar formalism - guided by the typing

Various Free constructions (eg extend the Gam construction to this poly-setting)

Robin

"Multiplicative Structure"

Representable polarized poly categories

TENSOR $\otimes$ "represents , on the left"

PAR $\oplus$ "represents , on the right"

So, eg, we have rules like

$$\frac{\Gamma_1, Q_1, Q_2, \Gamma_2 \vdash_0 \Delta_1 / Q \setminus \Delta_2}{\Gamma_1, Q_1 \otimes Q_2, \Gamma_2 \vdash_0 \Delta_1 / Q \setminus \Delta_2}$$

bijective actually

(and variants, for all possible positions of Q_1, Q_2 on the left)

(and binary $\otimes R$ rules to establish the "bijections")

[Typing remark: $Q \otimes Q$ is Q ; dually $P \oplus P$ is P]

Units (Eg) tensor unit T given by rules like

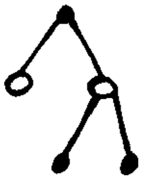
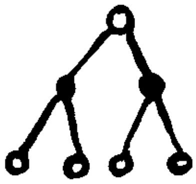
$$\frac{\Gamma_1, \Gamma_2 \underset{o\,p}{\mapsto} \Delta}{\Gamma_1, T, \Gamma_2 \underset{o\,p}{\mapsto} \Delta}$$

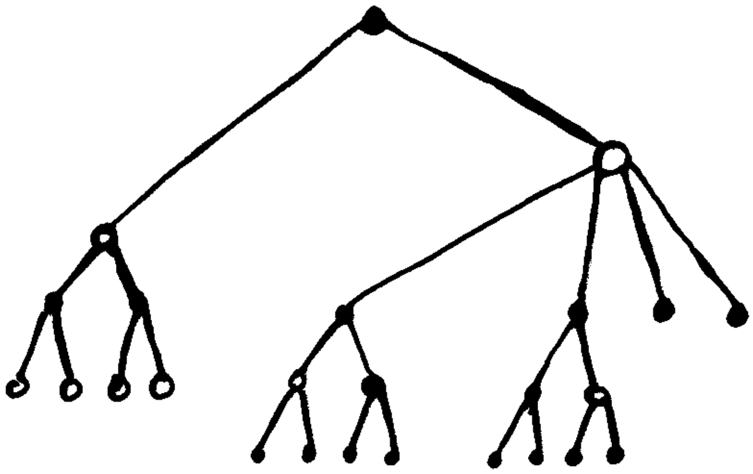
(+ variants, + TR rules to make this bijective)

(+ par unit $\perp$ dual)

[Typing: T is Q
$\perp$ is P]

Exercise :

 $\otimes$  $=$



BUT THERE'S MORE! ("Represent the context slashes")

Eg "represent" a $\backslash$ on the left:

$$\frac{\Gamma / P \backslash Q, \Gamma' \vdash_P \Delta}{\Gamma / P \,{<}\!\otimes\, Q \backslash \Gamma' \vdash_P \Delta}$$

Typing: $P \,{<}\!\otimes\, Q$ is P

There are 4 such "mixed" tensors, given by rules like the above (in all possible relevant positions, plus rules to make these bijective)

[Typing: $P \,{<}\!\otimes\, Q$ is P $\qquad$ $Q \,{<}\!\oplus\, P$ is Q
$Q \otimes\!{>}\, P$ is P $\qquad$ $P \oplus\!{>}\, Q$ is Q]

<u>Example</u>

The finite AJ games do carry all these 6 tensors/pars

so, eg,
$$Q \otimes Q' = (b_j : P_j \,{<}\!\otimes\, Q',\ b'_k : Q \otimes\!{>}\, P'_k \mid j \in J,\ k \in J')$$

where $Q = (b_j : P_j \mid j \in J)$, $Q' = (b'_j : P'_j \mid j \in J')$

<u>and</u>
$$P \,{<}\!\otimes\, Q = \{ a_i : Q_i \otimes Q \mid i \in I \}$$
$$Q \otimes\!{>}\, P = \{ a_i : Q \otimes Q_i \mid i \in I \}$$

where $P = \{ a_i : Q_i \mid i \in I \}$

Representability

{left-rep^n}

A string A, B is "dom-representable", by an object $A \otimes B$ if there are (polarized) (poly)natural bijections

$$\begin{array}{c} \Gamma, A, B, \Gamma' \underset{\circ\; p}{\mapsto} \Delta \\ \hline\hline \Gamma, A \otimes B, \Gamma' \underset{\circ\; p}{\mapsto} \Delta \end{array}$$

(+ the 3 variants where A, B can occur on the left in Q-positions)

The null string is "dom-representable", by an object $\top$ if

$$\begin{array}{c} \Gamma, \Gamma' \underset{\circ\; p}{\mapsto} \Delta \\ \hline\hline \Gamma, \top, \Gamma' \underset{\circ\; p}{\mapsto} \Delta \end{array}$$

(and 3 variants)

[Dually $\oplus$, $\bot$ are cod-representing objects]

{right-rep}

[And similarly for the context-slashes, represented by the mixed tensors]

A polarized polycategory is representable if all the above

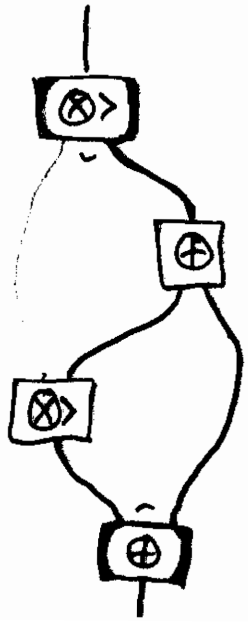
Linear Distributivity

Note : a representable pol. poly cat yields a "linear" polarized category structure by just taking arrows $X \to Y$ in $\underline{X}_o$, $\underline{X}_p$, $\hat{X}$

"Linear" in the sense that there are monoidal structures linked by linear distributivities (several, in fact – though at an appropriate 2-cat level these are components of appropriate polarized transformations)

Eg:

$$\dfrac{\dfrac{\dfrac{}{/B\oplus C\backslash \vdash_p B\oplus C}}{/B\oplus C\backslash \vdash_p B, C} \qquad \dfrac{\dfrac{}{/A\otimes\!> B\backslash \vdash_p A\otimes\!> B}}{A/B\backslash \vdash_p A\otimes\!> B}}{\dfrac{A/B\oplus C\backslash \vdash_p A\otimes\!> B, C}{/A\otimes\!>(B\oplus C)\backslash \vdash_p (A\otimes\!> B)\oplus C}}\ \text{CUT}$$



Note further: We get the necessary "coherence" <u>for free</u> (from the poly-naturality required of representability)

(So often it's easier to construct linear polarized categories via pol. <u>poly</u>categories + representability)

<u>Q</u> Can we <u>construct</u> (freely) representable structures over poly-structures?

- better, and add (or preserve) additive structure as well.

An easy eg: (without additives)

$$\text{Rep Pol Poly Cat} \xrightarrow{u} \text{Pol Poly Graph}$$

has a left adjoint (via circuits)

other egs .... Robin's Talk!

Other Topics

- Representability + free constructions (next! ...)
- Negation ($*$-autonomy in a polarized setting)
- Exponentials (finitary "Curien" ! - as in finitary AJ games)
- co-Kleisli construction
- "De-polarization" (Takes us back closer to the spirit of the original AJ games which were not explicitly polarized - we can get a $*$-aut cat)
- Laurent-style polarization

(CS : polarity based on "conjunction" vs "disjunction"
L : " " " "multiplicative" vs "additive")

Connection : via the Fam construction