is automatic: for any commutative diagram

$$\begin{tikzcd} X \arrow[r, "t"] \arrow[d, "r"'] & Y \arrow[d, "s"] \\ X' \arrow[r, "t'"'] & Y' \end{tikzcd}$$

and any formula φ over Y, there's a proof $\Sigma_r t^* \varphi \to t'^* \Sigma_s \varphi$ represented by the derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{rx = x' \wedge \varphi(tx)}{rx = x'}\,(\wedge\,\mathrm{E})}{t'rx = t'x'}\,(\mathrm{ap})}{stx = t'x'}\,(\mathrm{id})' \qquad \dfrac{rx = x' \wedge \varphi(tx)}{\varphi(tx)}\,(\wedge\,\mathrm{E})}{stx = t'x' \wedge \varphi(tx)}\,(\wedge\,\mathrm{I})}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{I}) \qquad \exists\xi(r\xi = x' \wedge \varphi(t\xi))}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{E})$$

(ap) is a derived rule: if $t\colon X \to Y$, $s_0, s_1\colon Z \to X$ are terms typed as indicated, (ap) is the derivation

$$\dfrac{s_0 = s_1 \qquad \dfrac{\top}{ts_0 = ts_0}\,(\mathrm{R})}{ts_0 = ts_1}\,(\mathrm{sub})$$

(id)′ is either the identity derivation (if st and $t'r$ are identical) or is a derived rule (if $(P)\colon \vdash st = t'r$ in T):

$$\dfrac{P(x) \qquad stx = t'rx \qquad t'rx = t'x'}{stx = t'x'}\,(\mathrm{T})$$

We give the inverse proofs for pullbacks of types (a), (b); note that type (a) actually gives two cases because of its lack of symmetry. Type (c) essentially follows from the isomorphism $\top_X \cong (x = x)$, in view of a later result, so we will not discuss it here. Also, the preservation of the Beck condition under products holds in the logic, so for the empty theory our job will be done with type (a) and (b) pullbacks. As for the situation for more general theories, this will be more fully discussed in § 8.

Type (a) (i). $\Delta^*_Y \sum_{t \times Y} \varphi \to \sum_t \langle X, t\rangle^* \varphi$: This is represented by the derivation (I) on p. 517. (Recall that the pair notation is a meta-notation, so that (pE), for example, is merely a special case of ($\wedge$ E): and $\exists\langle\xi, \eta\rangle\, \psi$ is really $\exists\xi\, \exists\eta\psi$. Hence the above use of ($\exists$ E), for example, actually indicates two such uses. This remark will also cover what follows.)

(I)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{