First, consider that part of P_1 above $rx_1 = x' \wedge \varphi(tx_1)$; writing Q_2 in full, it is easy to see that this is equivalent to the derivation

$$Q_2:\quad (\wedge\,\mathrm{I})\ \dfrac{(1)\quad(3)}{Q_2}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(1)\quad(3)}{Q_2}\,(\wedge\,\mathrm{I})}{x_1 = x}\,(\wedge\,\mathrm{E})}{rx_1 = rx}\,(\mathrm{ap}) \qquad \dfrac{\dfrac{(1)\quad(3)}{Q_2}\,(\wedge\,\mathrm{E})}{rx = x'}\,(\wedge\,\mathrm{E})}{rx_1 = x'}\,(\mathrm{T}) \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(1)\quad(3)}{Q_2}}{rx = x' \wedge \varphi(tx) \wedge x_1 = x}\,(\wedge\,\mathrm{I})}{x_1 = x}\,(\mathrm{ap})}{tx_1 = tx}\,(\mathrm{S})}{tx = tx_1} \quad \dfrac{rx = x' \wedge \varphi(tx)}{\varphi(tx)}\,(\wedge\,\mathrm{E})}{\varphi(tx_1)}\,(\mathrm{sub})}{rx_1 = x' \wedge \varphi(tx_1)}\,(\wedge\,\mathrm{I})$$

Let P_2 be the derivation P_1 with this subderivation replacing the subderivation above $rx_1 = x' \wedge \varphi(tx_1)$. Use (= Exp) and (= Simp) to get the equivalent derivation

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(1)}{rx = x' \wedge \varphi(tx)}}{rx = x'}\,(\wedge\,\mathrm{E})}{t'rx = t'x'}\,(\mathrm{ap})}{stx = t'x'}\,(\mathrm{id})}{[t'x' = stx]}\,(\mathrm{S})}{\begin{matrix}P_I\\ \exists\xi(r\xi = x' \wedge t\xi = tx)\end{matrix}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{T}_X}{x = x}\,(\mathrm{R})}{rx = rx}\,(\mathrm{ap}) \quad \dfrac{(1)}{rx = x'}\,(\wedge\,\mathrm{E})}{rx = x'}\,(\mathrm{T}) \qquad \dfrac{\dfrac{\dfrac{\mathsf{T}_X}{x = x}\,(\mathrm{R})}{tx = tx}\,(\mathrm{ap}) \quad \dfrac{(1)}{\varphi(tx)}\,(\wedge\,\mathrm{E})}{\varphi(tx)}\,(\mathrm{sub})}{rx = x' \wedge \varphi(tx)}\,(\wedge\,\mathrm{I})}{\exists\xi(r\xi = x' \wedge \varphi(t\xi))}\,(\exists\,\mathrm{I})}{\exists\xi(r\xi = x' \wedge \varphi(t\xi))}\,(\exists\,\mathrm{E})\ (2) \qquad \exists\xi(r\xi = x' \wedge \varphi(t\xi))}{\exists\xi(r\xi = x' \wedge \varphi(t\xi))}\,(\exists\,\mathrm{E})\ (1)$$

(We have used ($\wedge$ Red) to eliminate the derivation Q_2 above $rx = x'$.)

It is now easy to see that this is equivalent to the identity derivation, using (= Red), (= Exp), ($\wedge$ Exp), ($\exists$ E Simp), and ($\exists$ Exp).

(b) $\qquad t'^*\Sigma_s\varphi \to \Sigma_r t^*\varphi \to t'^*\Sigma_s\varphi = \mathrm{id}$.

We begin with the composite derivation (X) on p. 540.

(This time x_1 could be x—it will disappear of its own accord soon!) Using ($\exists$ Perm) (twice), ($\exists$ Red), ($\wedge$ Red) this is equivalent to the derivation

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(2)}{sy = t'x'}\,(\wedge\,\mathrm{E})}{[t'x' = sy]}\,(\mathrm{S})}{\begin{matrix}P_I\\ \exists\xi(r\xi = x' \wedge t\xi = y)\end{matrix}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(3)}{rx = x'}\,(\wedge\,\exists)}{t'rx = t'x'}\,(\mathrm{ap})}{stx = t'x'}\,(\mathrm{id})' \qquad \dfrac{\dfrac{\dfrac{(3)}{tx = y}\,(\wedge\,\mathrm{E})}{y = tx}\,(\mathrm{S}) \quad \dfrac{(2)}{\varphi(y)}\,(\wedge\,\mathrm{E})}{\varphi(tx)}\,(\mathrm{sub})}{stx = t'x' \wedge \varphi(tx)}\,(\wedge\,\mathrm{I})}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{I})}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{E})\ (3) \qquad \exists\eta(s\eta = t'x' \wedge \varphi(\eta))}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{E})\ (2)$$

By Corollary 1 of § 2 this is equivalent to

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(2)}{sy = t'x'}\,(\wedge\,\mathrm{E})}{[t'x' = sy]}\,(\mathrm{S})}{\begin{matrix}P_I\\ \exists\xi(r\xi = x' \wedge t\xi = y)\end{matrix}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(3)}{tx = y}\,(\wedge\,\mathrm{E})}{y = tx}\,(\mathrm{S}) \quad \dfrac{\dfrac{\dfrac{\dfrac{(2)}{sy = t'x' \wedge \varphi(y)}}{sy = t'x'}\,(\wedge\,\mathrm{E})}{t'x' = sy}\,(\mathrm{S})}{sy = t'x'}\,(\mathrm{S})}{stx = t'x'}\,(\mathrm{sub}) \qquad \dfrac{\dfrac{\dfrac{(3)}{tx = y}\,(\wedge\,\mathrm{E})}{y = tx}\,(\mathrm{S}) \quad \dfrac{(2)}{\varphi(y)}\,(\wedge\,\mathrm{E})}{\varphi(tx)}\,(\mathrm{sub})}{stx = t'x' \wedge \varphi(tx)}\,(\wedge\,\mathrm{I})}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{I})}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{E})\ (3) \qquad \exists\eta(s\eta = t'x' \wedge \varphi(\eta))}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{E})\ (2)$$

Using (= Exp) and (= Simp), this is equivalent to

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(2)}{sy = t'x'}\,(\wedge\,\mathrm{E})}{[t'x' = sy]}\,(\mathrm{S})}{\begin{matrix}P_I\\ \exists\xi(r\xi = x' \wedge t\xi = y)\end{matrix}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{T}_Y}{y = y}\,(\mathrm{R}) \quad \dfrac{\dfrac{\dfrac{\dfrac{(2)}{sy = t'x' \wedge \varphi(y)}}{sy = t'x'}\,(\wedge\,\mathrm{E})}{t'x' = sy}\,(\mathrm{S})}{sy = t'x'}\,(\mathrm{S})}{sy = t'x'}\,(\mathrm{sub}) \qquad \dfrac{\dfrac{\mathsf{T}_Y}{y = y}\,(\mathrm{R}) \quad \dfrac{(2)}{\varphi(y)}\,(\wedge\,\mathrm{E})}{\varphi(y)}\,(\mathrm{sub})}{sy = t'x' \wedge \varphi(y)}\,(\wedge\,\mathrm{I})}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{I})}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{E})\ (3) \qquad \exists\eta(s\eta = t'x' \wedge \varphi(\eta))}{\exists\eta(s\eta = t'x' \wedge \varphi(\eta))}\,(\exists\,\mathrm{E})\ (2)$$