expansion for $\vee$ and $\exists$.)

It is now evident how Π has the structure of a 2-category Π : its objects are formulae, its morphisms are derivations, its 2-cells are the operations mentioned above. (We indicate domains and codomains for permutations and expansions; the rest should be obvious - just insert " $\Rightarrow$ " in the proper blank spaces in [P1] , § II.3.3.1 .) The definitions of compositions and identities are canonical.

We have lax 2-functors $\&$, $\vee$: $\Pi \times \Pi \to \Pi$, where "lax" is defined in § 2(i) . For example, $\&(A,B) = A \& B$. Also, there is a diagonal 2-functor $\Delta : \Pi \to \Pi \times \Pi$, $\Delta(A) = (A,A)$. (The quantifiers $\exists$, $\forall$ can also be considered as lax 2-functors between suitable 2-categories, and there is a suitable "diagonal" of opposite sense. Implication is best considered as a lax 2-functor $A \supset (\ )$: $\Pi \to \Pi$, for fixed A .)

The following meta-principle ("reduction" for operations) is useful:

(R) An expansion of an occurrence of a logical symbol, followed by a reduction of the same occurrence, is (provided the composite is an endooperation) the identity operation.

2. Suppose we are given the following data:

(i) $\mathcal{A}$, $\mathcal{B}$ are 2-categories,

$$\mathcal{A} \underset{G}{\overset{F}{\rightleftarrows}} \mathcal{B}$$

are lax 2-functors, in the sense that instead of strict functoriality for morphisms, we have "comparison 2-cells":

$$\gamma^F_{gf} : FgFf \Rightarrow F(gf) \quad , \quad \iota^F_A : 1_{FA} \Rightarrow F1_A \quad ,$$

for $A \xrightarrow{f} B \xrightarrow{g} C$ in $\mathcal{A}$. (G similarly.)

(ii) For any $A \in \mathcal{A}$, $B \in \mathcal{B}$, there are functors

$$(FA,B) \underset{\lambda_{AB}}{\overset{\kappa_{AB}}{\rightleftarrows}} (A,GB)$$

(κ , λ will be made "lax 2-natural transformations" in ways specified below.)

DEFINITION 1. Suppose, for any $A' \xrightarrow{f} A$ in $\mathcal{A}$, $B \xrightarrow{g} B'$ in $\mathcal{B}$ there are natural transformations $k_{fB} : \kappa_{A'B}(Ff,B) \Rightarrow (f,GB)\kappa_{AB}$,
$k_{A'g} : \kappa_{A'B'}(FA',g) \Rightarrow (A',Gg)\kappa_{A'B}$, $\ell_{fB} : (Ff,B)\lambda_{AB} \Rightarrow \lambda_{A'B}(f,GB)$,
$\ell_{A'g} : (FA',g)\lambda_{A'B} \Rightarrow \lambda_{A'B'}(A',Gg)$, $\alpha_{AB} : 1_{(FA,B)} \Rightarrow \lambda_{AB}\kappa_{AB}$,
$\beta_{AB} : \kappa_{AB}\lambda_{AB} \Rightarrow 1_{(A,GB)}$.