$\vee$ -permutation

$$\begin{array}{c} \dfrac{\begin{array}{c}\Sigma_0\\ A_1 \vee A_2\end{array} \quad \begin{array}{c}[A_1]\\ \Sigma_1\\ B\end{array} \quad \begin{array}{c}[A_2]\\ \Sigma_2\\ B\end{array}}{[B]} \\ \Sigma_3 \\ C \end{array} \quad \Longrightarrow \quad \dfrac{\begin{array}{c}\Sigma_0\\ A_1 \vee A_2\end{array} \quad \begin{array}{c}[A_1]\\ \Sigma_1\\ [B]\\ \Sigma_3\\ C\end{array} \quad \begin{array}{c}[A_2]\\ \Sigma_2\\ [B]\\ \Sigma_3\\ C\end{array}}{C}$$

(provided the RHS is a derivation)

$\exists$ -permutation

$$\begin{array}{c} \dfrac{\begin{array}{c}\Sigma_0\\ \exists\, x\, Ax\end{array} \quad \begin{array}{c}[A(a)]\\ \Sigma_1\\ B\end{array}}{[B]} \\ \Sigma_3 \\ C \end{array} \quad \Longrightarrow \quad \dfrac{\begin{array}{c}\Sigma_0\\ \exists\, x\, Ax\end{array} \quad \begin{array}{c}[A(a)]\\ \Sigma_1\\ [B]\\ \Sigma_3\\ C\end{array}}{C}$$

(provided the RHS is a derivation)

We remark that (given the reductions) the existence of Prawitz' $\vee$-expansion and $\vee$-permutation is equivalent to the existence of the following form of $\vee$-expansion:

$$\begin{array}{c} \Sigma_0 \\ [A \vee B] \\ \Sigma_1 \\ C \end{array} \quad \Longrightarrow \quad \dfrac{\begin{array}{c}\Sigma_0\\ A \vee B\end{array} \quad \begin{array}{c}\dfrac{[A]}{[A \vee B]}\\ \Sigma_1\\ C\end{array} \quad \begin{array}{c}\dfrac{[B]}{[A \vee B]}\\ \Sigma_1\\ C\end{array}}{C}$$

(And analogously for $\exists$; we shall henceforth use these stronger forms of